Mixed-Frequency Predictive Regressions*

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Abstract

This paper explores the performance of mixed-frequency predictive regressions for stock returns from the perspective of a Bayesian investor. We develop a parameter learning approach for sequential estimation, allowing for belief revisions. Empirically, we find that mixed-frequency models improve predictability, not only because of the combination of predictors with different frequencies but also due to the preservation of the time-variation in the volatility of predictors. Mixed-frequency models produce higher volatility timing benefits, compared to temporally aggregate models. Therefore, our results highlight the importance of consistently incorporating predictors of mixed frequencies and correctly specifying the volatility dynamics in predictive regressions.

JEL Classification: C11, C32, C53, G11

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1 Introduction

A large body of literature demonstrates that stock returns are predictable by various financial and macroeconomic variables. Among the most widely examined financial predictors are asset valuation ratios.[1] Typically, they capture stock return variations from the cash flow perspective. Financial predictors also include measures of equity risk, with the predictability stemming from their linkage to time-varying equity premiums. At the same time, there is evidence that macroeconomic variables such as, e.g., the consumption-wealth ratio as introduced by Lettau and Ludvigson (2001), predict stock returns over the business cycle frequency. Hence, it is natural to ask whether a combination of both financial and macroeconomic predictors produces superior forecasts of stock returns.[2]

However, financial predictors are usually available at a monthly frequency, whereas we observe macroeconomic predictors on a quarterly or yearly basis, following macroeconomic announcements. The availability of these data at different frequencies hampers their combination to form predictions. Often, direct treatment of such data is circumvented by aggregating the data with a higher frequency, thereby reducing all data to the lowest frequency. Such a procedure, however, does not fully exploit all available information in the data. On the one hand, it may smooth out statistically significant high-frequency features such as time-varying stock return volatility, further giving rise to biased statistical inference.[3] On the other hand, high-frequency features can also be economically critical, and ignoring them is costly. For example, Moreira and Muir (2017) and many others find that portfolio rebalancing according to changes in volatility is profitable.

In this paper, we propose a dynamic Bayesian estimation framework for predictive regressions that explicitly accounts for the presence of mixed-frequency predictors. The advantage of our setup is that, in contrast to the commonly used aggregation procedure, it respects the high-frequency features of financial predictors, such as stochastic volatility. Our estimation approach builds on the particle learning method of Carvalho, Johannes, Lopes, and Polson (2010), given its well-known

[1]The most commonly used asset valuation ratios are the dividend-price ratio, earning-price ratio, and dividend-earning ratio; see, e.g., Campbell and Shiller (1988a), Campbell and Shiller (1988b), Cochrane (1992), Ang and Bekaert (2006), Cochrane (2007), Lettau and Van Nieuwerburgh (2007), and Cochrane (2011).

[2]See, for example, Rapach, Strauss, and Zhou (2010).

[3]For example, Johnson (2018) argues that predictive regressions accounting for time-varying volatility produce better forecasts.

advantage of incorporating an ensemble of statistically and economically essential features such as sequential parameter estimation, time-varying volatility, state latency, and estimation risk. We take particle learning as our starting point and extend it to a mixed-frequency setup with the assumption of nonnegative expected returns. As argued by Campbell and Thompson (2008), such a constraint improves model forecasts.

To illustrate the effect of incorporating low-frequency macroeconomic predictors into predictive regressions with monthly evolving variables, we consider the consumption-wealth ratio (CAY) proposed by Lettau and Ludvigson (2001) as our candidate macroeconomic variable. CAY is only available at a quarterly frequency. For the monthly predictors, we rely on the set of variables used in Welch and Goyal (2007). As our goal is to focus on the incremental effect of combining a low-frequency with a high-frequency predictor, we restrict our analysis to bivariate regressions.[4] We measure the economic performance of predictability by the portfolio gains of an investor who allocates her wealth to the aggregate stock and the risk-free asset by maximizing expected utility. The investor is assumed to be Bayesian in the sense that she does not observe the parameters, but learns about the stock return dynamics sequentially and takes all sources of uncertainty into account. Particle learning, compared to other approaches, is ideally suited to model such an investment problem as it allows us to mimic the belief revision of the investor.

By comparing models with and without monthly-evolving stochastic volatility (SV) and CAY, we find that both features significantly improve return forecasts. In particular, the evolution of the degree of predictability by CAY echoes the slow regime shifts of interest rates, asset valuations, and risk premiums documented by Bianchi, Lettau, and Ludvigson (2016). Moreover, incorporating SV improves model inference, raises predictability, and significantly reduces parameter uncertainty. Economically, SV models produce significant portfolio gains through both the enhanced return forecast and volatility timing. The forecasting power of CAY translates itself into portfolio gains consistently in time. Summarizing our results, we find that both SV and CAY significantly raise the average portfolio return, Sharpe ratio, and certainty equivalent return. Through volatility timing, SV also

[4]In principle, one could use our framework to analyze predictive regressions with multiple predictors. Also, instead of using CAY to predict stock returns, one might think of different applications, e.g., using quarterly available survey data on inflation and exchange rates as low-frequency predictor. See, e.g., Foroni et al. (2015) and, for an overview, Foroni and Marcellino (2013).

improves portfolio higher-order moments such as skewness and excess kurtosis.

Lastly, we explore whether it makes a difference if we explicitly account for mixed frequencies or if we follow a naïve approach and aggregate the data to the lowest frequency. In particular, we compare our mixed-frequency models with quarterly aggregate models in which we assume that the volatility is also quarterly-evolving. We find that the quarterly SV-CAY model severely misspecifies the evolution frequency of the volatility dynamics and consequently produces worse portfolio higher-order moments. The quarterly SV model even underperforms the quarterly constant-volatility model, indicating that the cost of misspecification is economically significant.

Our work is related to at least two streams of literature. The first stream is concerned with the mixed-frequency econometric analysis[5] and Bayesian econometrics, both of which have attracted considerable attention over the last years. A commonly used frequentist approach is the mixed-data sampling regression proposed by Ghysels, Santa-Clara, and Valkanov (2004). Another branch takes a Bayesian view, among which the works most relevant to ours are Schorfheide and Song (2015) and Schorfheide, Song, and Yaron (2018). They embed a mixed-frequency particle filter in an MCMC iterator that accounts for parameter inference, whereas our extension builds on the real-time parameter learning method of Carvalho, Johannes, Lopes, and Polson (2010) and Johannes, Korteweg, and Polson (2014). The advantage of our approach is that it simultaneously incorporates an ensemble of model features including sequential parameter estimation, estimation risk, state latency, mixed-frequency data, and economic constraints.

The second stream of literature related to our work is concerned with stock return predictability. The predictive power of CAY of is firstly examined by Lettau and Ludvigson (2001). Together with various financial predictors, it is further studied, among others, by Welch and Goyal (2007). They find poor in-sample and out-of-sample predictability when using a simple regression framework. Johannes, Korteweg, and Polson (2014) confirm these results. However, considering a Bayesian investor and incorporating estimation risk and stochastic volatility, they find statistically significant performance improvements for the dividend-yield ratio and net payout yield. We go beyond their frame-

[5]For some of the most recent contributions in this field, we refer to the special issue on mixed-frequency data analysis of the Journal of Econometrics, Volume 193, Issue 2, Pages 291-446 (August 2016). The role of parameter learning and estimation risk, without taking into account the role of mixed frequencies, is examined by Brennan (1998), Stambaugh (1999), Barberis (2000), Xia (2001), Brandt, Goyal, Santa-Clara, and Stroud (2005), Johannes, Korteweg, and Polson (2014), and many others.

work and further identify the benefit when we enrich the predictive regressions with low-frequency macroeconomic predictors. Moreover, using our approach we can preserve high-frequency features like stochastic volatility, which is crucial to exploit potential benefits from volatility timing.[6] In addition, Johnson (2018) argues that predictive regressions with stochastic volatility produce greater predictability.

The remainder of this paper is organized as follows. Section 2 formulates the mixed-frequency predictive regressions. Section 3 outlines the constrained mixed-frequency particle learning setup and conducts model specification analysis. Section 4 examines the portfolio management implications of models with SV and CAY. Section 5 concludes.

2 Predictive Regressions

In specifying the predictive regressions, most of the literature considers stock returns and predictors sampled at the same frequency. If low-frequency predictors are used, the common practice is to aggregate stock returns to the same sampling frequency. Here, we generalize the predictive regressions by consistently combining predictors that are available at different frequencies.

2.1 Data Description

For examining stock return predictability, we use the dataset from Welch and Goyal (2007).[7] The dataset covers the dividend-price ratio (DP), dividend yield (DY), earning-price ratio (EP), dividend-payout ratio (DE), stock variance (SVAR), book-to-market ratio (BM), net equity expansion (NTIS), treasury-bill rate (TBL), long-term yield (LTY), long-term rate of return (LTR), term spread (TMS), default yield spread (DFY), default return spread (DFR), and inflation (INFL). Due to the delayed announcement, we use INFL with one more lag to predict stock returns. DP, DY, EP, and DE are measured in logarithmic terms. These variables capture different aspects of stock return variations and fall into several categories: (i) asset valuation ratios, such as DP, DY EP, DE, and BM; (ii) measures

[6] Indeed, there is ample evidence that volatility timing provides additional economic benefits for portfolio strategies, see, for example, Fleming, Kirby, and Ostdiek (2001), Fleming, Kirby, and Ostdiek (2003), Johannes, Korteweg, and Polson (2014), Gargano, Pettenuzzo, and Timmermann (2017), and Moreira and Muir (2017).

[7] The sample set covers all data used in this paper and is available at Amit Goyal's web page: http://www.hec.unil.ch/agoyal/.

of equity risk including SVAR and NTIS; (iii) measures of bond yield characteristics including TBL, LTY, LTR, TMS, DYS, and DFR; and finally (iv) INFL. These variables are usually used to predict stock returns at a monthly frequency. Using monthly data is also consistent with the fact that portfolios are often rebalanced on a monthly basis.

Among the literature of macroeconomic predictors for stock returns, Lettau and Ludvigson (2001) examined the transitory deviation of the shared trend of logarithmic consumption (c), asset holdings (a), and labor income (y), defined by

$$\mathrm{CAY}_t = c_t - \beta_a a_t - \beta_y y_t, \tag{1}$$

where β_a and β_y are coefficients determining the cointegration relation among these variables. They find that the trend deviation summarizes the predictive component of the consumption-wealth ratio of the aggregate economy and christen it as CAY. The rationale for its predictive power is that, given high expected returns, investors may increase consumption to maintain a smooth consumption stream over time, thereby raising CAY. Thus, it may also capture stock return variations over the business cycle frequency that are empirically not reflected in accounting-based asset valuation ratios.[8]

CAY is not directly observable, and concerns regarding its estimation naturally emerge. First of all, CAY is extracted from macroeconomic variables that are only quarterly sampled most of the time in history. Second, the original construction of CAY by Lettau and Ludvigson (2001) relies on the full-sample estimates of β_a and β_y, and is thus not suitable for evaluating the out-of-sample performance of predictive regressions. Recently, Bianchi, Lettau, and Ludvigson (2016) empirically document a low-frequency shift of CAY, which, however, can be difficult to identify out of sample, particularly for investors who learn about stock return predictability and rebalance portfolios on a monthly basis. We thus follow Welch and Goyal (2007) and use the sequential out-of-sample estimate of CAY

$$\mathrm{CAY}_t = c_t - \widehat{\beta}_{a,t} a_t - \widehat{\beta}_{y,t} y_t, \tag{2}$$

[8] The forecasting power of CAY has been widely examined in the literature; see Cochrane (2011), Pástor and Stambaugh (2012), Kostakis, Magdalinos, and Stamatogiannis (2014), Bianchi, Lettau, and Ludvigson (2016), Hsu, Palomino, and Qian (2017), Johnson (2018), etc.

where $\widehat{\beta}_{a,t}$ and $\widehat{\beta}_{y,t}$ are estimated from all data available up to each quarter t. Other macroeconomic variables may also predict stock returns and could be included for a more extensive study. The prime goal of this paper, however, is to examine the potential benefit from enriching monthly predictive regressions with low-frequency, macroeconomic variables. Thus, we exclusively focus on univariate monthly and quarterly predictors. As the time series of CAY extends from January 1952 to December 2016, we use monthly stock returns and financial predictors in the same period. In Table 1, we report the summary statistics of the stock returns (including dividends), risk-free rate, monthly predictors, and quarterly CAY.

[Table 1 about here.]

2.2 Model Setup

The standard univariate predictive regression uses a lagged monthly-sampled variable, denoted by Z_t, to forecast next month's excess stock return

$$r_{t+1}^{\text{ex}} = K_{r,0} + K_{r,1} Z_t + \sigma_r \epsilon_{t+1}^r, \tag{3}$$

where r_{t+1}^{ex} is the logarithmic monthly stock return in excess of the risk-free rate, ϵ_{t+1}^r is the forecasting error and i.i.d. standard normally distributed, $K_{r,0}$ and $K_{r,1}$ are coefficients determining the forecasted return, and σ_r is the standard deviation of the forecasting error. Imposing the constraint $K_{r,1} = 0$ immediately gives the normal model with constant mean and variance. This degenerate specification assumes that stock returns are not predictable, and is often used as the benchmark against predictors. If the stock return is predictable by Z_t, the estimated expected return fluctuates with Z_t. The predictor thus allows investors to "time" future returns.

The issue of mixed sampling frequencies arises when we attempt to incorporate low-frequency predictors such as, e.g., the quarterly CAY, into the monthly-evolving predictive regression in equation (3). Traditional approaches often involve temporally aggregating monthly variables to quarterly frequency. Such a simplification, however, does not respect the mixed-frequency nature of the available data and often incurs an information loss. For predicting stock returns, the cost of temporal aggregation can be particularly severe, as statistically significant and economically important high-frequency features such as time-varying volatility are very likely to be smoothed out, thereby giving rise to biased inference and economic losses for investors who manage portfolios based on predictability and volatility timing. There is thus a need for models that allow mixed-frequency predictors while, at the same time, preserving these high-frequency features.

To achieve this goal, we integrate mixed-frequency predictors and various model features including the time-varying volatility in the spirit of the Harvey accumulator (Harvey, 1990), which turns out convenient in the dynamic Bayesian framework. More precisely, we disaggregate CAY into a monthly-evolving latent process X, i.e.,

$$\mathrm{CAY}_{t+3} = X_{t+1} + X_{t+2} + X_{t+3}, \tag{4}$$

where we assume X to follow a monthly-evolving linear-Gaussian process of the form

$$X_{t+1} = K_{X,0} + K_{X,1} X_t + \sigma_X \epsilon^X_{t+1}, \tag{5}$$

with ϵ^X_{t+1} i.i.d. standard normally distributed. Our assumption of the linear-Gaussian model (5) is motivated by the stylized empirical fact that CAY is persistent and mean-reverting. CAY is observed only at the end of each quarter, whereas X is not observed. Incorporating CAY as a macroeconomic predictor is fulfilled by using lagged values of X to forecast the stock return in the next month:[9]

$$r^{\mathrm{ex}}_{t+1} = K_{r,0} + K_{r,1} Z_t + K_{r,2} X_t + \sigma_r \epsilon^r_{t+1}. \tag{6}$$

There is a sufficient degree of freedom in specifying the dynamics of X. The motivation for using equation (4) is twofold. First, aggregating equation (6) to quarterly frequency gives a form that is highly similar, though not identical, to a quarterly predictive regression, which we will analyze in Section 4.3. Second, equation (4) can be interpreted as a log-linear approximation, up to a scale

[9] The mixed-frequency predictive regression presumes that CAY is observed, with X to be estimated. A more sophisticated specification involves parameterizing the cointegration relation in equation (1), which is jointly estimated with equations (3) to (6). However, improvements of model inference and economic gains from this model is not significant. Thus, we follow the convention and simply use the estimated CAY for predicting stock returns.

change, to an arithmetic average of CAY that preserves the linear structure of the state-space model.[10]

At first glance, the mixed-frequency models in equations (3) to (6) are closely connected to the imperfect predictive systems of Pástor and Stambaugh (2009). Indeed, embedding our setup into theirs, X determines the expected return and CAY plays the role of the imperfect predictor. They argue that predictability depends on the correlation between unexpected returns and innovations in the expected returns. To explore the role of correlation, we also estimate models that allow a constant correlation between ϵ^r_{t+1} and ϵ^X_{t+1}. The sequential estimate fluctuates around -0.1 and is not statistically significant at 90% level consistently over time, thereby ruling out the predictive role of the correlation coefficient. Further, our empirical findings suggest that predictability stems from CAY or X itself rather than the correlation to their innovations. The wedge between our finding and Pástor and Stambaugh (2009) is likely because the CAY is only quarterly observed, which makes the correlation coefficient much less identifiable at a monthly frequency. In contrast, the predictive regression with CAY employed by Pástor and Stambaugh (2009) is aggregated to a quarterly frequency. We thus set the correlation between stock returns and CAY to zero throughout the rest of this paper.

In light of the importance of time-varying volatility discussed before, we also consider stochastic volatility models, which take the form

$$r^{\text{ex}}_{t+1} = K_{r,0} + K_{r,1} Z_t + K_{r,2} X_t + \sqrt{V_t}\epsilon^r_{t+1}, \tag{7}$$

with the monthly instantaneous variance V_t assumed to follow a log-linear Gaussian process

$$\ln V_{t+1} = K_{V,0} + K_{V,1} \ln V_t + \sigma_V \epsilon^V_{t+1}, \tag{8}$$

where ϵ^V_{t+1} are i.i.d. standard normal. The parameters $K_{V,0}$ and $K_{V,1}$ capture the persistent and mean-reverting nature of the instantaneous variance. σ_V determines the volatility of volatility. The correlation $\rho = \text{Corr}(\epsilon^r_{t+1}, \epsilon^V_{t+1})$ captures the volatility leverage effect. Empirically, the correlation is

[10] Incorporating mixed-frequency data by the sum aggregation, as formulated in equation (5), is common in the macroeconomic literature and has been adopted by Mariano and Murasawa (2003), Aruoba, Diebold, and Scotti (2009), Schorfheide and Song (2015), Marcellino, Porqueddu, and Venditti (2016), Schorfheide, Song, and Yaron (2018), to name a few.

significantly negative and thus allows investors to identify a high volatility state when the market drops suddenly.[11] We find that the log-Gaussian specification is capable of generating volatility spikes and outperforms other specifications such as the wide class of time-varying volatility models studied, e.g., by Christoffersen, Jacobs, and Mimouni (2010). Therefore, for our empirical analysis, we focus on the volatility specification in equation (8) when referring to a stochastic volatility model.

To summarize, the predictive regressions considered in this paper nest four classes of models: univariate constant volatility (CV) and stochastic volatility (SV) models with one monthly predictor only, and CV and SV models with CAY as an additional macroeconomic predictor, labeled CV-CAY and SV-CAY models. When there is a particular emphasis on the monthly financial predictor, for example, DP, we use CV(SV)-DP and CV(SV)-DP-CAY to denote models under consideration. We also consider all specifications without any monthly predictor, labeled CV(SV)-C and CV(SV)-C-CAY models, respectively, where C denotes "constant". The predictive regressions are thus either univariate or bivariate with mixed frequencies. In total, we have 60 different models. The rich dataset and model specifications allow us to explore the statistical and economic gains from CAY and SV, and their robustness to monthly predictors.

While the novelty of our setup is that it preserves high-frequency model features such as stochastic volatility when low-frequency predictors are incorporated, we acknowledge that other features beyond our setup can also be important. For instance, there is evidence of time variation in parameters in the literature.[12] However, our empirical findings in Section 3.4 suggest that the time-varying volatility of stock returns can greatly resolve the time variation and uncertainty of parameter estimates. Furthermore, while Johannes, Korteweg, and Polson (2014) favor models in which the volatility of the monthly predictor is also time-varying, we refrain from introducing time-varying variance in the dynamics of CAY. Given the quarterly frequency of CAY, a monthly-evolving stochastic volatility process would be only weakly identifiable. Hence, we restrict our analysis to the most parsimonious setup given by equations (3) to (8), which nevertheless enables us to explore the benefits from incorporating model features available at mixed frequencies.

[11] There is a wide consensus on the volatility leverage effect at daily or higher frequency. Johannes, Korteweg, and Polson (2014) argue that ρ is not statistically significant at a monthly frequency and set it as zero. In contrast, we find that sequential estimate of ρ takes a value between -0.4 and -0.3 and is significantly distinct from zero.

[12] See, for example, Paye and Timmermann (2006), Lettau and Van Nieuwerburgh (2007), Henkel, Martin, and Nardari (2011), Dangl and Halling (2012), and Johannes, Korteweg, and Polson (2014).

3 Parameter Learning

For estimation, we employ the sequential parameter learning approach proposed by Carvalho, Johannes, Lopes, and Polson (2010). The sequential nature provides great convenience for analyzing out-of-sample predictability in a real-time fashion. Economically, sequential estimation of the CV model in equations (5) and (6) and the SV model in equations (5), (7) and (8), mimics the dynamic belief revision of an investor who believes that the data-generating process is summarized by one of these models. The investor is assumed to be Bayesian, in the sense that she does not observe the parameters or state variables, but learns about them sequentially in time as new observations arrive, following Bayes' rule. The belief of the investor is represented by the joint distribution of parameters and state variables. In the standard setup of Carvalho, Johannes, Lopes, and Polson (2010), parameter learning is achieved by tracking sufficient statistics of parameter distributions. We extend their methodology to account for mixed-frequency predictors and the assumption of nonnegative expected returns. The economic constraint, as argued by Campbell and Thompson (2008), improves the forecasting performance.

3.1 Particle Learning

We use Y_t to denote the collection of all observations in month t. Therefore, we have $Y_t = (r_t^{\text{ex}}, Z_t)$, or $Y_t = (r_t^{\text{ex}}, Z_t, \text{CAY}_t)$ whenever t is the last month of a given quarter. Furthermore, we denote the set of all parameters by Θ. To account for the quarterly frequency of CAY, the filter to be presented requires augmenting the state space by one more lag of X. The state vector at time t is thus $L_t = (X_{t-1:t}, V_t)$, where $X_{t-1:t} = (X_{t-1}, X_t)$. Let $Y^t = Y_{1:t} = (Y_1, Y_2, ..., Y_t)$ be the collection of all observations available up to time t. Given Y^t, the investor belief is fully summarized by the joint posterior of state variables and parameters, which we denote by $p(L_t, \Theta|Y^t)$.

Particle learning mimics the belief revision in a fully adapted manner. Technically speaking, given Y_{t+1}, particle learning infers $p(L_{t+1}, \Theta|Y^{t+1})$ directly from $p(L_t, \Theta|Y^t)$, which makes out-of-sample analysis of predictability and portfolio performance feasible. The general idea is to update the parameter posterior by tracking its sufficient statistics, which we denote by s_t. Particle learning views the sufficient statistics as additional state variables that drive the parameter posterior and embeds them into a particle filter. To retain analytical tractability in parameter updating, particle learning starts with a conjugate prior. By definition, a conjugate prior makes the posterior belong to the same distribution family. Therefore, the parameter posterior at each time is determined by the same type of sufficient statistics, but with values updated by observations and filtered state variables.

We briefly outline the machinery of particle learning in this section and defer the details to Appendix A. By Bayes' rule, the joint posterior $p(L_{t+1}, s_{t+1}|Y^{t+1})$ can be obtained from $p(L_t, s_t, \Theta|Y^t)$ via[13]

$$\begin{aligned} p\left(L_{t+1}, s_{t+1} \middle| Y^{t+1}\right) \quad \propto \quad & \int_{(L_t, s_t, \Theta)} p\left(Y_{t+1}, \middle| L_t, \Theta, Y_t\right) p\left(L_{t+1} \middle| L_t, \Theta, Y_{t:t+1}\right) \\ & \cdot p\left(s_{t+1} \middle| L_{t:t+1}, s_t, Y_{t:t+1}\right) dp\left(L_t, s_t, \Theta \middle| Y^t\right). \end{aligned} \tag{9}$$

where $p(Y_{t+1}, |L_t, \Theta, Y_t)$ is the predictive likelihood, $p(L_{t+1}|L_t, \Theta, Y_{t:t+1})$ is the posterior of the state variable at time $t+1$, and $p(s_{t+1}|L_{t:t+1}, s_t, Y_{t:t+1})$ summarizes the sufficient statistics updating rule given observations $Y_{t:t+1}$ and state variables $L_{t:t+1}$. Because the predictive regression uses lagged variables to forecast future returns, all above terms are jointly determined by Y_t and Y_{t+1}. Since s_{t+1} is the collection of conditional sufficient statistics of Θ given Y^{t+1}, we can obtain the joint posterior directly from

$$p\left(L_{t+1}, s_{t+1}, \Theta \middle| Y^{t+1}\right) = p\left(L_{t+1}, s_{t+1} \middle| Y^{t+1}\right) p\left(\Theta \middle| s_{t+1}\right). \tag{10}$$

Particle learning overcomes the lack of analytic tractability in the presence of joint parameter uncertainty and state latency by representing all distributions by their sample draws

$$\widehat{p}\left(L_t, s_t, \Theta \middle| Y^t\right) = \sum_{i \in \mathcal{I}} \mathbb{I}_{(L_t, s_t, \Theta)^{(i)}}, \tag{11}$$

where i denotes the i-th sample and $\mathcal{I}$ is the set of indices. Propagating from time t to $t+1$ employs an importance-sampling procedure with weights proportional to the predictive likelihood, and with

[13]Our formulation builds on the auxiliary particle filter of Pitt and Shephard (1999) and employs a resample-propagate scheme. The extension to mixed-frequency setup is convenient and equivalent to a forward smoother used to deal with lagged state variables. The forward smoother is examined by Kitagawa (1994) for particle filters.

state variables at time $t+1$ drawn from their filtering densities. Thanks to the conditional normality, the likelihood function in any month, whenever CAY is observed, can be evaluated explicitly for all model specifications. The state density is also conditionally normal and provides convenience for sample draws.

3.2 Economic Constraint

The traditional risk-return relation suggests that the equity premium should be nonnegative. Campbell and Thompson (2008) find that predictive regressions accounting for the nonnegativity constraint produce more favorable forecasting performance. Along with this line of research, Pettenuzzo, Timmermann, and Valkanov (2014) and Chib and Zeng (2016) propose Bayesian estimators that account for the nonnegativity constraint. More specifically, Pettenuzzo, Timmermann, and Valkanov (2014) merely rely on an acceptance-rejection scheme to retain the nonnegativity, while Chib and Zeng (2016) adopt the weaker assumption of a sign restriction on the Bayesian predictive mean instead of the entire posterior of the expected return. They derive a truncated forecast identical to Campbell and Thompson (2008) but go one step further by obtaining the parameter posterior associated with the truncated forecast. The posterior is econometrically well-grounded as it minimizes the entropy for the naïve Bayesian posterior among all candidates satisfying the above nonnegativity constraint.

We embed the nonnegativity constraint into particle learning by combining their methods. In this section, we briefly outline our approach, but delegate all details to Appendix A. For each Monte Carlo sample, we use an acceptance-rejection scheme in which parameter draws determining the expected return (K_r) are rejected until the expected return associated with this sample becomes non-negative. A potential drawback is that if the Bayesian predictive mean is negative, the acceptance-rejection scheme may take too many draws to proceed. To alleviate this concern, we draw parameters from the minimum entropy posterior. The minimum entropy posterior pushes the Bayesian predictive mean above zero and thus gives an acceptance rate larger than 50%. The sampling efficiency is thus significantly improved. Analogously to Pettenuzzo, Timmermann, and Valkanov (2014), another advantage of our approach is that it exploits the economic constraint not only for forecasting but also in estimation.

3.3 Prior Specification

Implementing the particle learning algorithm requires the initialization of the joint prior of the state variables and sufficient statistics.[14] We construct a noninformative prior by forming the point estimates and sufficient statistics merely from data that extend from January 1952 to December 1961.

We construct the uninformative prior in a way that is highly consistent with each model specification. Specifically, to estimate the regression coefficients $(K_{r,0}, K_{r,1}, \sigma_r^2)$ for CV models we use the standard ordinary least square (OLS) regression. For SV models, we switch to the generalized least square (GLS) to account for heteroscedasticity. Monthly observations are used exclusively for both specifications. For CV-CAY models, we estimate the parameters $(K_{r,0}, K_{r,1}, K_{r,2}, \sigma_r^2)$ by aggregating all observations to quarterly frequency.

For SV-CAY models, we estimate the parameters $(K_{r,0}, K_{r,1}, K_{r,2})$ by GLS with temporal aggregation. To estimate the parameters governing the variance dynamics, we exclude CAY and obtain the OLS residuals by regressing monthly stock returns on the monthly predictor only. We use the squared OLS residual as an approximation of the realized variance. Using monthly variables only and excluding CAY is motivated by the observation that a time-varying variance is typically much less pronounced at low sampling frequencies such as quarterly. We further estimate the variance parameters by the first-order autoregression of the log-squared residual. For the estimation of the correlation ρ, we use the residuals of the monthly GLS regression and the monthly first-order autoregression of variance.

Lastly, we estimate the parameters governing X by firstly estimating the first-order autoregression of CAY, and then disaggregating it to monthly frequency. The instantaneous variance in the last month of the training set, $V_{1961:\mathrm{M}12}$, is sampled from its stationary distribution. Because CAY is highly persistent, $X_{1961:\mathrm{M}12}$ is simulated by assuming that the previous state is $\mathrm{CAY}_{1961:\mathrm{Q}4}/3$. Furthermore, since the regression-based estimate is a rather crude guess of the true parameters, we set the degree of freedom to 12, i.e., we view the prior as being trained from one-year data. We use

[14] Since r^{ex} and L essentially take a conditional linear-Gaussian form, the conjugate priors for their dynamics are thus normal or normal-inverse-gamma. For the correlation ρ, we use the specification of the conjugate prior utilized in Johannes, Korteweg, and Polson (2014). The sufficient statistics governing the posterior can be updated using observations and state variables represented by sequential Monte Carlo samples.

the period from January 1962 to December 1971 as the burn-in period for particle learning. The sample size of each subperiod is large enough, allowing us to form a stable estimate. We base our analysis of different model specifications and portfolio management implications exclusively on the sample period January 1972 to December 2016.

3.4 Sequential Estimates

We find that incorporating CAY into CV and SV models only marginally affects the estimates of parameters controlling monthly predictors and volatility. Moreover, the dynamics of CAY and volatility are well estimated and are similar for models with different monthly predictors. Therefore, we focus on the coefficients controlling the forecasted returns, namely, $K_{r,0}$, $K_{r,1}$, and $K_{r,2}$, whose posterior mean and standard errors are displayed in Tables 2 and 3. The posterior mean and standard errors are obtained in the last month of the sample period.

[Table 2 about here.]

[Table 3 about here.]

To display sequential parameter estimates, we take DP as an illustrating example as it is the most well-known asset valuation ratio with its forecasting power widely examined by the literature.[15] In Figure 1, we report at each time t the posterior mean and the 90%-credible interval, defined as the area between 5% and 95% quantiles of parameter posteriors. We repeat this exercise in each month to form the sequential estimates for the period from January 1972 to December 2016.

[Figure 1 about here.]

The forecasting power of DP and CAY is linked to the estimates of $K_{r,1}$ and $K_{r,2}$, respectively, as they quantify how much DP and CAY can explain future stock return. The width of the credible bands captures the amount of uncertainty, and the time variation of point estimates and credible bands captures the instability of parameter estimates. Panels A and B of Figure 1 display their

[15] See, for instance, Campbell and Shiller (1988a), Campbell and Shiller (1988b), Cochrane (1992), Ang and Bekaert (2006), Cochrane (2007), Lettau and Van Nieuwerburgh (2007), and Cochrane (2011).

estimates sequentially in time for both CV-DP-CAY and SV-DP-CAY models. First, there is evidence of declining stock return predictability by both DP and CAY, especially for the CV-DP-CAY model. Panel A shows that the estimate of $K_{r,1}$ is statistically significant in the first half of the sample period. However, as time evolves, the estimate tapers off, particularly in the late 1990s, and becomes eventually insignificant.[16] Second, CAY produces significant improvement in stock return predictability. In Panel B, the sequential estimate of $K_{r,2}$ is statistically significant most of the time.

Figure 1 also reveals strong evidence of short-term parameter shifts and misspecification for the CV-DP-CAY model. For instance, the estimate of $K_{r,1}$ exhibits a significant variation particularly during the 1973 oil crisis, the 1987 stock market crash, and the late 1990s. For all parameter estimates of the CV-DP-CAY model in Figure 1, the amount of uncertainty measured by the size of the 90%-credible interval dissipates slowly over time. This observation suggests that the predictability is unstable and that learning about it is slow even after 20 years' training (1952 - 1971). A similar pattern is observed for CAY, as illustrated by Panel B. In particular, we observe in Panel C of Figure 1 that the posterior mean of the variance σ_r^2 increases sharply in all periods of market turmoils including the 1973 oil crisis, the 1987 stock market crash, the burst of the dotcom bubble, and the 2008 Great Recession. The variance estimate is thus inconsistent with the assumption of constant variance and suggests that CV models are misspecified. Figure 2 compares the point estimate of the instantaneous variance for CV-DP-CAY and SV-DP-CAY models, defined as the posterior mean of the variance filtered from stock returns. It is clear that the instantaneous variance given by the SV-DP-CAY model varies substantially over time and thus casts considerable doubts on CV models.

[Figure 2 about here.]

In Panels A and B of Figure 1, we also observe that incorporating SV significantly improves return predictability and model inference.[17] For the SV-DP-CAY model, the sequential estimate of $K_{r,1}$ becomes more stable, larger in the second half of the sample period, and eventually converges to the estimate of $K_{r,1}$ for the CV-DP-CAY model. The estimate of $K_{r,2}$ becomes consistently larger over

[16] Our sequential estimate of $K_{r,1}$ demonstrates a similar pattern to the estimate in Johannes, Korteweg, and Polson (2014).

[17] Aside from the sequential estimates, we also examined other measures of model performance. The results are summarized as follows: SV models produce a larger joint predictive likelihood of stock return than CV models. CAY gives a larger predictive likelihood that is relevant to forecasting stock returns, in the sense of Geweke and Amisano (2010). Moreover, CV-CAY models produce larger out-of-sample R^2 than CV models.

the entire sample period, which implies that predictability by DP, and CAY in particular, becomes stronger for SV models. Moreover, the sequential estimates of $K_{r,1}$ and $K_{r,2}$ exhibit less variation with narrower 90%-credible intervals, implying that incorporating SV considerably reduces both the uncertainty and instability in model estimation. The estimation efficiency comes from the fact that SV models overweight (underweight) the importance of observations when the signal-to-noise ratio is high (low), where the signal contains information about the expected return and the noise is proportional to return variance. In finite samples, such an advantage can be particularly significant compared to CV models, as argued by Johannes, Korteweg, and Polson (2014). Empirically, our results are consistent with their findings and Johnson (2018): predictive regressions accounting for stochastic volatility improve the forecasting power of DP, CAY, and other variables.

For other predictors similar findings are revealed by Tables 2 and 3. Under SV specifications, both monthly predictors and CAY exhibit greater forecasting power, with a larger magnitude of estimated coefficients and smaller standard errors, followed by larger statistical significance. There are only a few exceptions, including SVAR for both CV and CV-CAY models. Overall, the improvement in terms of forecasting power from incorporating SV is pervasive.

A particularly exciting phenomenon regarding CAY, as illustrated in Panel B of Figure 1, is the apparent structure breaks of the estimate of $K_{r,2}$ in 1976 and 2001. In the beginning, the estimate trends up rapidly, which we find is robust to different ways of training the prior. Then, we observe only a slow weakening for the forecasting power of CAY, but this weakening has intensified since 2001. The slow time variation of the estimated $K_{r,2}$ coincides with the decade-lasting regime shifts documented by Bianchi, Lettau, and Ludvigson (2016). They identify the period 1976 to 2001 as with low interest rates and asset valuations and high market risk premiums. Consistently, our empirical results indicate that predictability as quantified by $K_{r,2}$ is also stronger within this period. Ideally, models with CAY shall also account for the regime switching, but practically, this would not work as such a slow change in predictability cannot be identified within such a limited sample period.

From unreported estimates for the dynamics of CAY and volatility, we find that our models capture their empirical dynamics reasonably well. All parameters exhibit no substantial short-term shifts. The estimate of the volatility leverage effect captured by the correlation between stock returns and shocks to log variance is significantly negative, with the posterior mean consistently lying between

-0.4 and -0.3, as indicated by Panel D of Figure 1. Thus, ignoring the leverage effect only leads to biased estimates of other parameters and the instantaneous volatility.

Summarizing our empirical estimates, we conclude that both monthly-varying SV and quarterly CAY are crucial features for predicting stock returns. In particular, time-varying variance improves predictability through both the expected return coefficients and volatility. Moreover, the uncertainty of parameter estimates that persists in the entire sample period motivates us also to encompass estimation risk for portfolio management.[18] Although there is evidence on the time variation of parameters in the literature, our findings indicate that most of the short-term time variation comes from parameters governing the expected return which, however, significantly weakens when SV is incorporated. Therefore the reduction of the time variation of parameter estimates, as we observe in Panels A and B of Figure 1, can be viewed as an additional attribute of SV models. This finding is also what motivates us to place more emphasis on the statistical and economic impact of SV and CAY, rather than to add additional model complexity with additional time-varying parameters.

4 Bayesian Portfolio Management

The previous section demonstrates the predictive role of SV and CAY. We next examine the out-of-sample portfolio performance of a Bayesian investor who exploits these model features. Consistent with the previous estimation results, SV and CAY produce significant excess portfolio gains. In particular, we find that the portfolio gains from the mixed-frequency setup relative to the temporally aggregate models are economically significant.[19]

[18] The portfolio management implication of parameter learning and estimation risk is significant and examined by, for example, Brennan (1998), Stambaugh (1999), Barberis (2000), Xia (2001), and Johannes, Korteweg, and Polson (2014).

[19] For the following results, we also performed a series of robustness checks. First, we examined different sample periods and hyperparameters (e.g., the degree of freedom of the prior) for training the prior and portfolio performance assessment. We also considered Bayesian inference without the constraint of nonnegative expected returns and different relative risk-aversion rates. While the portfolio performance and holdings vary accordingly, the economic gains, measured by the difference of portfolio performance measures given by different model specifications, remain significant.

4.1 Portfolio Optimization

Following Pettenuzzo, Timmermann, and Valkanov (2014) and many others, we assume that at each month t the investor forms the optimal portfolio by maximizing the one-month expected utility

$$\max_{w_t \in [-2,3]} \mathbb{E}_t\big[U(R_{t+1,w})\big|Y^t\big], \tag{12}$$

where by $R_{t+1,w}$ we denote the gross return of the strategy that invests a fraction, denoted by w_t, of the total wealth into the aggregate stock at time t

$$R_{t+1,w} = (1-w_t)R^f_{t+1} + w_t R_{t+1}. \tag{13}$$

R^f_{t+1} and R_{t+1} are the gross return of the risk-free asset and the aggregate stock from time t to $t+1$, respectively. Like Johannes, Korteweg, and Polson (2014), among many others, we restrict our portfolio weights such that $w_t \in [-2,3]$, which implies that short selling and borrowing are allowed, but that the positions do not become too extreme. We equip our investor with a power utility function with risk-aversion rate γ,

$$U\left(R_{t+1,w}\right) = \frac{R^{1-\gamma}_{t+1,w}}{1-\gamma}. \tag{14}$$

We furthermore assume a myopic investment strategy and impose an investment horizon of only one month. Such a setup is sufficient for our purpose, which is to illustrate how and whether predictability and volatility timing materialize in portfolio benefits. The expected utility in equation (12) is evaluated according to the time-t belief of the investor, summarized by the joint posterior of state variables and parameters. For each fixed w_t, the expected utility is calculated using the sample representation of the posterior, which is available from the particle learning algorithm proposed in Section 3:

$$\begin{aligned}
\mathbb{E}_t\big[U\left(R_{t+1,w}\right)\big|Y^t\big] &= \int \mathbb{E}_t\big[U\left(R_{t+1,w}\right)\big|L_t,\Theta\big]\,dp\left(L_t,\Theta\big|Y^t\right) \\
&\approx \int \mathbb{E}_t\big[U\left(R_{t+1,w}\right)\big|L_t,\Theta\big]\,d\widehat{p}\left(L_t,\Theta\big|Y^t\right) \\
&\approx \frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}} \mathbb{E}\left[U\left(R_{t+1,w}\right)\Big|(L_t,\Theta)^{(i)}\right].
\end{aligned} \tag{15}$$

Given each draw from the posterior $(L_t, \Theta)^{(i)}$, we simulate the return $R_{t+1,w}^{(i)}$ and the realized utility $U\left(R_{t+1,w}^{(i)}\right)$ as an approximation of $\mathbb{E}\left[U(R_{t+1,w})\middle|(L_t,\Theta)^{(i)}\right]$. The expected utility is further obtained by marginalizing over all parameters and state variables. The optimal portfolio is obtained by solving the utility maximization problem (12).[20] This stock position is then held for one month, with gains and losses realized at time $t+1$. The belief is then revised given new observations at time $t+1$. This procedure is repeated until the end of the entire period.

It is noteworthy that the Bayesian framework accounts for parameter and state uncertainty, which affects the portfolio choice through variance and higher-order moments. For example, in the absence of parameter and state uncertainty, the monthly return is conditionally normal, and variance risk is the only source of uncertainty. When parameter and state uncertainty is incorporated, the excess return variance follows the decomposition

$$\mathrm{Var}_t\left(r_{t+1}^{\mathrm{ex}}\right) = \mathbb{E}_t\left[\mathrm{Var}_t\left(r_{t+1}^{\mathrm{ex}}\middle|L_t,\Theta\right)\right] + \mathrm{Var}_t\left(\mathbb{E}_t\left[r_{t+1}^{\mathrm{ex}}\middle|L_t,\Theta\right]\right), \tag{16}$$

where the first component summarizes the sequential point estimate of return variance while the second component summarizes the uncertainty of the expected return estimate. Thus, estimation risk also accounts for return variance. Further, by marginalization, parameter and state uncertainty produces return skewness and kurtosis. Thus, even for CV models, the effect of higher-order moment risk can be nontrivial. This setup differs from the frequentists' practice where the decision making is solely based on the point estimates of return mean and variance. Power-utility investors do care about higher-order moments, and SV models capture the nonnormality, especially the kurtosis, better than CV models, as empirically confirmed by Johannes, Korteweg, and Polson (2014). Considering this fact, we also explore the higher-order moments when analyzing portfolio returns.

[20] The power utility has a strictly positive support on which it is strictly concave and monotonously increasing. Allowing short selling and borrowing results in potentially negative realized or simulated wealth processes. For utility maximization, we bound the simulated gross return above 1%. The portfolio weight w_t is optimized over a grid of $[-2, 3]$ with a step size of 0.001. The whole procedure can be implemented efficiently. In our implementation, the realized wealth process given by the optimal portfolio remains strictly positive. Aside from such a computational convenience, using simulations avoids the integrability issue that arises in calculating the expected utility in the Bayesian framework. For a discussion, see Kandel and Stambaugh (1996) and Johannes, Korteweg, and Polson (2014). We also consider $w_t \in [0, 1]$, which is equivalent to forbidding short selling and borrowing. Our empirical conclusions remain unchanged. Further, different relative risk-aversion rates such as $\gamma = 2, 6$, and 8 are considered for robustness checks, with similar conclusions drawn from the portfolio performance.

4.2 Portfolio Performance

We base our empirical analysis of the portfolio strategies on the period January 1972 to December 2016, which gives 540 monthly portfolio returns. To obtain a first impression on the performance of the different model specifications, we plot the wealth gains averaged over different financial predictors when we use constant and stochastic volatility and when using additionally the quarterly CAY as predictor.

In Figure 3, we plot the evolution of the log wealth averaged across models with different monthly predictors but the same specification. We note that the average across the CV models underperforms the market slightly. Switching to an average SV model improves the performance considerably, which is in line with the findings in Johannes et al. (2014). However, the average of SV models is outperformed by average of CV models if we add CAY as an additional predictor to the latter. Clearly, the best performing model is the average SV model when CAY is included.

[Figure 3 about here.]

To get some further insights into the characteristics of the different strategies, we calculate a set of performance measures, including the average log excess return, its standard deviation, skewness, excess kurtosis, the Sharpe ratio, and the certainty equivalent return (CER). As in Johannes et al. (2014), we define the annualized CER of a trading strategy w based on simple returns as

$$\mathrm{CER}(t,T) = 12 \cdot \left[\left(\frac{\sum_{t=1}^{T} U\left(R_{t,w}\right)}{\sum_{t=1}^{T} U\left(R_t^f\right)} \right)^{\frac{1}{1-\gamma}} - 1 \right]. \tag{17}$$

Intuitively, the CER is the risk-free excess logarithmic return that gives the same cumulative realized utility as the trading strategy w. For each of the performance figure, we test whether it differs significantly across different models. For that purpose, we use the test statistics of Ledoit and Wolf (2018), which can be generally applied to smooth functions of population means of the underlying returns and is robust to non-normality and serial dependence.[21]

[21]For the test statistics, we use for the bootstrap procedure the algorithm of Ledoit and Wolf (2018) to choose data-dependent block sizes. For more details, we refer to their paper. In Table 4, when comparing with the SV-CAY models, we use '*' to indicate significance, when comparing CV and SV models, we use '$^\diamond$'. In Table 5, we use again '*'

[Table 4 about here.]

Table 4 reports the performance figures for the SV and CV strategies without using CAY. Additionally, we include two benchmark strategies, a simple buy-and-hold strategy (BH) and the prevailing-mean model (PM), which assumes that stock returns are i.i.d. normal and that portfolio optimization takes no estimation risk into account. Comparing Panel A and B, we find that, compared to their CV counterparts, all SV models produce a higher average portfolio return, Sharpe ratio, and CER. For eight (eight) models, the average return (Sharpe ratio) difference is statistically significant, while for the CER, all differences are highly significant at the 1% level, except for the CV-C model. Hence, an SV specification is strongly favored, at least when we include a monthly predictor. This finding is in line with Johannes et al. (2014). Indeed, we find that the average SV model has higher mean and Sharpe ratio than the average CV model at the 5% confidence level, and a higher CER at a 1% significance level. The differences in standard devation, skewness, and kurtosis are statistically insignificant.[22]

Clearly, our analysis in Table 4 is similar to that in Johannes et al. (2014). However, it is based on a different time period and a broader set of financial predictors, not just dividend yields, and some interesting differences emerge. While Johannes et al. (2014) conclude that the SV-C model is outperformed by their SV model including dividend yields as predictor, our results in Panel B do not support such a conclusion in general. Although, we find that five SV models with predictors significantly outperform the SV-C model in terms of CER,[23] there are also five SV models significantly underperforming the SV-C at the 5% confidence level. Moreover, for the Sharpe ratios, we do not find any significant differences that survive the robustness test of Ledoit and Wolf (2018). Therefore, while SV tends to enhance return predictability as documented in Section 3.4, we do not find evidence that this improved predictability unambiguously leads to economic gains as suggested by previous literature.

In a next step, we ask whether an ensemble of both financial and macroeconomic factors may

when comparing against SV-CAY in Panel A. In Panel B, use $^\bullet$ and $^\circ$ when comparing against the benchmark models PM and BH. Also, to test for CER differences, we compare the mean values of CER(t, s) defined in equation (17), for s from t to T. We do not present all statistical tests and model comparisons in these tables, but they can be obtained from the authors.

[22] These results can be obtained by the authors on request.

[23] We do not report these test statistics here, but they can be obtained from the authors.

generate statistically significant portfolio improvements. In Table 5, we present the results when we enrich the univariate predictive regressions with CAY as a low-frequency macroeconomic predictor. Comparing first CV-CAY with the results of CV from Table 4, we find that adding CAY not only increases the mean excess return but also its standard deviation. Nevertheless, CV-CAY produces favorable Sharpe ratios for all predictors. However, in unreported results, we find that the differences in the Sharpe ratio are insignificant at the 5% level, except for DFR as predictor. Since adding CAY considerably increases kurtosis, the CV-CAY specification in terms of CER becomes less favorable. Only for four predictors, the CER difference between CV-CAY and CV is significantly positive at the 5% level. For six predictors it is significantly negative. Hence, adding CAY to a constant volatility model does not yet convincingly improve portfolio performance, neither in terms of Sharpe ratios nor in terms of CER.

[Table 5 about here.]

In Table 5, Panel A, we also report the significance tests when comparing CV-CAY with SV. While there are no notable differences in terms of mean excess returns, there are significant differences in terms of standard deviation, negatively impacting both Sharpe ratios and CERs. However, in terms of Sharpe ratio we do not find significant differences. However, since CV-CAY has much larger kurtosis and higher (negative) skewness, the CER of the SV strategies are significantly larger at the 1% level. Hence, we can expect that when we move from CV-CAY to SV-CAY, we can further improve portfolio performance. Indeed, as Panel B of Table 5 shows, the Sharpe ratios become significantly larger for five predictors. With less negative skewness and lower kurtosis, the CER becomes significantly larger for all predictors at the 1% level. Hence, similarly to our previous observation in Table 4, the incorporation of SV is crucial to significantly improve portfolio performance.

When comparing SV-CAY with SV, we find that adding CAY leads to higher mean excess returns, Sharpe ratios, and CER. From these increases in mean returns, nine of them are statistically significant. Although only two of the increases in the Sharpe ratio are significant due to the simultaneous increase in standard deviation, the comparison in terms of CER is convincingly favorable for the SV-CAY specification with 13 significantly positive differences. For two models (NTIS and TMS), the difference is negative but insignificant.

Lastly, we include in Panel B of Table 5 the results of the significance tests when comparing SV-CAY against the two benchmark models. It turns out that SV-CAY significantly increases the portfolios mean excess returns, at the expense of a higher standard deviation. However, the SV-CAY models generate a decrease in negative skewness, which eventually leads to superior CER. Compared to the buy-and-hold strategy BH, all CERs are statistically significantly higher. Except for two predictors (BM and NTIS), the same holds true when comparing against the PM strategy, mostly due to its relatively low standard deviation. Also in terms of Sharpe ratios, the SV-CAY models significantly outperform the two benchmarks, except when BM is used as predictor. To conclude, SV-CAY models significantly outperform not only the benchmarks, but also the SV and CV-CAY models. Hence, the joint incorporation of SV and a low-frequency predictor like CAY is an economically essential model feature.[24]

[Figure 4 about here.]

As an additional analysis, we calculate the alpha of the different strategies relative to market. In particular, we regress the portfolio's excess returns on the excess return of the market. In Figure 4, we plot the resulting cumulative alphas. From Panel A, we observe that all the CV models generate a negative alpha over the period ranging from January 1972 to December 2016. The best performing strategy is the CV-C model, which generates a cumulative alpha that stays close to zero for most of the time. In Panel B, the CV-CAY models are able to provide some positive cumulative alphas, but they reverse during the 2008 financial crises and enter negative territory towards the end of the period. In contrast, in Panel C, SV models are still able to produce positive alphas after the financial crisis. While all strategies end up in positive territory, some cumulative alphas were still negative before the financial crisis. Interestingly, the SV-C model performs similarly well as the average SV model using a financial predictor. In Panel D, we plot the cumulative alphas of the SV-CAY models. They all generate positive alphas for (almost) the whole period.

[Table 6 about here.]

[24] As an additional exercise, we also analyze a strategy that allocates equal weights to the strategies with different predictors. Again, we find that the averaged SV-CAY provides significant positive differences both in terms of Sharpe ratios and CER, when compared to the average CV, SV, and CV-CAY. These results can be obtained by the authors.

To assess the statistical significance of the observations in Figure 4, we use the robust method of Leippold and Rüegg (2018).[25] In Table 6, we report the alpha estimates and their corresponding p-values. While none of the CV models, as expected, produces any significantly positive alphas, we find four significant alphas from CV-CAY and eleven significant alphas from SV models. For SV-CAY, all 15 models provide significant alphas, eleven of them at the 1% significance levels, ranging up to an annualized alpha of 4.365%. Clearly, adding CAY to SV provides an extra edge in terms of performance relative to the market.

4.3 Economic Gains from Mixed-Frequency Specifications

Conventional approaches to deal with mixed frequencies involve temporally aggregating high-frequency variables, for example, time-varying volatility, to the lowest frequency. Intuitively, such a naive aggregation should result in some performance loss. However, it is not a-priori clear how large such a loss will be and whether it is statistically significant. For our analysis, we start with the quarterly model where only CAY is used to forecast stock returns at a quarterly horizon

$$r^{\mathrm{ex}}_{t,t+3} = K_{r,0} + K_{r,1}\mathrm{CAY}_t + \sigma_r \epsilon^r_{t,t+3}, \tag{18}$$

where t is the last month of a given quarter, $r^{\mathrm{ex}}_{t,t+3}$ is the quarterly log excess return, and the forecasting error $\epsilon^r_{t,t+3}$ is assumed i.i.d. standard normal. The model has constant variance and is thus called Q-CV-C-CAY model, where we use C to emphasize that there is no monthly predictor and Q to denote a quarterly frequency. We also examine the quarterly stochastic volatility model, labeled Q-SV-C-CAY and taking the form of

$$r^{\mathrm{ex}}_{t,t+3} = K_{r,0} + K_{r,1}\mathrm{CAY}_t + \sqrt{V_t}\epsilon^r_{t,t+3}, \tag{19}$$

where V_t follows the same specification as equation (8) but is quarterly-evolving. We use exactly the same treatment to train the prior.

[25] This method also relies on block-resampling to account for serial correlation. For the block size, we use the method of Politis and White (2004) and its correction in Patton et al. (2009), and for each test we use 50,000 bootstrap repetitions.

To allow a fair evaluation of mixed-frequency models, we compare CV-C-CAY and SV-C-CAY models with the above quarterly-evolving model. We consider an investor who rebalances her position only quarterly, i.e., in the last month of each quarter, denoted by t, the investor chooses a portfolio weight by maximizing the expected utility in the next quarter

$$\max_{w_t \in [-2,3]} \mathbb{E}_t \left[U\left(R_{t,t+3,w}\right) | Y^t \right], \tag{20}$$

with $R_{t,t+3,w}$ denoting the quarterly return of strategy w,

$$R_{t,t+3,w} = (1 - w_t) R^f_{t,t+3} + w_t R_{t,t+3}, \tag{21}$$

and $R^f_{t,t+3}$ and $R_{t,t+3}$ denoting the quarterly gross returns of the risk-free asset and the aggregate stock, respectively. The investor has a power utility with risk aversion rate γ

$$U\left(R_{t,t+3,w}\right) = \frac{R^{1-\gamma}_{t,t+3,w}}{1-\gamma}, \tag{22}$$

and the wealth dynamics follows $W_{t+3} = W_t \cdot R_{t,t+3,w}$. To put the models with different evolution frequencies on a comparable basis, we let investor determine the portfolio weight at time t and holds the portfolio until the end of the next quarter, with gains and loss realized at time $t+3$. For an investor who learns monthly but trades quarterly, multi-horizon forecasts of monthly predictors are required to obtain the quarterly predictive return distribution, which further requires a parametric model for monthly predictors. To retain model parsimony, we switch off the monthly predictor Z and exclusively focus on SV and CAY. Comparing mixed-frequency models with quarterly aggregate models is informative about the economic gain from a mixed-frequency specification.

Regarding the sequential estimates of CV and SV models, we draw the same conclusion as in the previous sections. For both monthly and quarterly models, SV enhances the forecasting power of CAY and improves model inference in various dimensions. What makes monthly and quarterly models distinct is the estimate of the volatility dynamics, which we analyze in more detail below.

[Figure 5 about here.]

Figure 5 compares the sequential estimates of parameters controlling the volatility dynamics for SV-C-CAY and Q-SV-C-CAY models, including $K_{V,0}$, $K_{V,1}$, σ_V^2, and ρ, and shows that a quarterly-evolving volatility model is subject to severe misspecification. First of all, there is a significantly larger short-term and long-term variation of estimates of all parameters for the quarterly model than for the monthly model. This finding suggests either the estimate converges more slowly at a quarterly frequency in finite samples or the constant parameter specification is insufficient. Second, there is a smaller amount of uncertainty in the estimates of the mixed-frequency model, as the 90%-credible interval is narrower. An explanation is that monthly frequency provides a data-rich environment for model inference, which potentially reduces the uncertainty of parameter estimates.

Second, time-varying volatility is a high-frequency phenomenon by its nature and thus can be better inferred from monthly returns than lower-frequency, for example, quarterly data. Figure 6 supports this claim. It illustrates the instantaneous variance filtered according to SV-C-CAY and Q-SV-C-CAY models. By comparison, the monthly model filters out a rapid volatility movement at a higher frequency especially in periods of high volatility. For example, during the 2008 financial crisis, the monthly model identifies a sharp increase in volatility which drops afterward and quickly bounces back, whereas the quarterly model is confronted with a significant delay, and fails to identify such a rapid change that occurs in a single quarter. Thus, learning based on the quarterly model is subject to severe estimation risk.

[Figure 6 about here.]

We then compare the portfolio gains from learning based on the mixed-frequency and quarterly aggregate models. The portfolio optimization exercise results in 180 quarterly samples of portfolio returns. In Table 7, we report the portfolio performance for a risk-aversion rate of $\gamma \in \{2, 4, 6, 8\}$. As a first test, we check whether the resulting portfolio weights differ significantly between SV-C-CAY and Q-SV-C-CAY. We find that they do and that their difference is highly significant at the 1% level, with larger weights for the SV-C-CAY model that correctly incorporates the mixed frequencies. In terms of mean excess return, the SV-C-CAY model outperforms the Q-SV-C-CAY model, but it also generates a higher standard deviation due to the larger portfolio weight. Hence, although the SV-C-CAY model generates higher Sharpe ratios, they do not differ significantly. However, since

SV-C-CAY also exhibits a less negative skewness and a lower kurtosis, all the CERs are significantly higher at the 1% level. Lastly, we can calculate the resulting alphas of SV-C-CAY and Q-SV-C-CAY relative to the market. Although its alpha is positive, Q-SV-C-CAY fails to generate a significant alpha. In contrast, the alphas of SV-C-CAY are significant at the 5% level.

[Table 7 about here.]

To illustrate the evolution of the portfolio wealth and the impact of consistently incorporating mixed frequencies, we plot in Figure 7, Panel A, the evolution of the log returns of SV-C-CAY and Q-SV-C-CAY and compare in with the market. While both models outperform the market, the SV-C-CAY model does so by a large margin. A dollar invested in January 1972, the market would return 45 dollars by the end of 2016, 62 dollars if we follow Q-SV-C-CAY, and 135 dollars if we follow the SV-C-CAY. These differences are also reflected in Panel B of Figure 7, where we plot the cumulative alpha of the two strategies with respect to the market. The size of the cumulative alpha of SV-C-CAY further highlights the importance of consistently incorporating predictors of mixed frequencies and correctly specifying the volatility dynamics in predictive regressions.

[Figure 7 about here.]

5 Conclusion

There is ample evidence that stock returns are predictable by financial fundamentals, such as valuation metrics and measures of equity risk, and macroeconomic variables. The literature documents an empirical improvement from combining these predictors but mostly ignores the fact that macroeconomic predictors are potentially available at lower frequencies, for example, quarterly. Traditional solutions involve temporally aggregating high-frequency variables such as stock returns and financial predictors to lower frequency, but tend to smooth out high-frequency features. However, these high-frequency features, such as volatility, can be statistically and economically significant, and ignoring them leads to biased inference and severe opportunity costs of investors who exploit the predictability.

Considering this tradeoff, we propose a unified framework that simultaneously incorporates mixed-frequency model features or predictors in predictive regressions. We develop a sequential Bayesian estimator that accounts for mixed frequencies and economic constraints. The sequential algorithm also models the dynamic belief updating of a Bayesian investor who learns about stock return predictability. As an illustrating example, we incorporate the quarterly consumption-wealth ratio (CAY) into monthly predictive regressions with stochastic volatility (SV). We find that both SV and CAY are statistically and economically essential model features.

We not only explore the role of SV and CAY but also analyze the benefit from such a mixed-frequency setup. We find that the quarterly aggregated SV-CAY model misspecifies the evolution frequency of the volatility dynamics, resulting in poor volatility timing and giving worse portfolio performance than the mixed-frequency specification. Thus, our results highlight the importance of incorporating low-frequency macroeconomic predictors while preserving a high-frequency volatility specification.

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A Constrained Mixed-Frequency Particle Learning

The appendix presents the constrained mixed-frequency particle learning algorithm for SV-CAY models. Switching off SV or CAY trivially gives particle learning algorithms for CV, CV-CAY, and SV models.

A.1 Model Specification

Recall that SV-CAY models take the form

$$\begin{aligned}
r^{\text{ex}}_{t+1} &= K_{r,0} + K_{r,1} Z_t + K_{r,2} X_t + \sqrt{V_t}\epsilon^r_{t+1}, \\
X_{t+1} &= K_{X,0} + K_{X,1} X_t + \sigma_X \epsilon^X_{t+1}, \\
\ln V_{t+1} &= K_{V,0} + K_{V,1} \ln V_t + \sigma_V \epsilon^V_{t+1}, \qquad \text{(A.1)}
\end{aligned}$$

with $\text{Cov}(\epsilon^r_{t+1}, \epsilon^V_{t+1}) = \rho$. Denote the collection of all parameters by Θ

$$\Theta = \Big\{ K_r, K_X, \sigma_X, K_V, \sigma_V, \rho \Big\}, \qquad \text{(A.2)}$$

where $K_r = (K_{r,0}, K_{r,1}, K_{r,2})$, $K_X = (K_{X,0}, K_{X,1})$, and $K_V = (K_{V,0}, K_{V,1})$. We assume that the priors for all parameters are conjugate. To be more precise, we set the prior of K_r to be normal, and that of (K_X, σ^2_X) and (K_V, σ^2_V) to be normal-inverse-gamma. The prior of ρ follows the specification of Johannes, Korteweg, and Polson (2014) and will be presented latter. The sufficient statistics of each prior and posterior are analytically tractable and can be updated sequentially due to their conjugate nature. Furthermore we define Y_t as the collection of all observations in month t, with $Y^t = Y_{0:t} = (Y_1, Y_2, ..., Y_t)$. $L_t = (X_{t-1:t}, V_t)$ is the vector of latent variables at time t. $s = (s^r, s^X, s^V, s^\rho)$ is the collection of all sufficient statistics.

A.2 Particle Learning

We present steps to implement particle learning. We approximate the joint posterior $p(L_t, s_t, \Theta | Y^t)$ with $N = 50,000$ Monte Carlo samples, denoted by

$$\widehat{p}\left(L_t, s_t, \Theta \middle| Y^t\right) = \sum_{i \in \mathcal{I}} \mathbb{I}_{(L_t, s_t, \Theta)^{(i)}}, \tag{A.3}$$

where $\mathcal{I} = \{1, 2, ..., N\}$ is the sample index. Particle learning takes the following steps to move from time t to $t+1$.

(i). Draw N indices $(n^{(i)})_{i \in \mathcal{I}}$ from $\mathcal{I}$ according to the weights

$$w_t^{(i)} \propto p\left(Y_{t+1} \middle| (L_t, \Theta)^{(i)}, Y_t\right), \quad i \in \mathcal{I}, \tag{A.4}$$

where $n^{(i)}$ is the index of the i-th draw.

(ii). Draw $L_{t+1}^{(i)}$ from the conditional joint density

$$L_{t+1}^{(i)} \sim p\left(L_{t+1} \middle| (L_t, \Theta)^{(n^{(i)})}, Y_{t:t+1}\right). \tag{A.5}$$

(iii). Update the sufficient statistics for each sample

$$s_{t+1}^{(i)} = \mathcal{S}\left(s_t^{(n^{(i)})}, (L_t, \Theta)^{n^{(i)}}, L_{t+1}^{(i)}, Y_{t:t+1}\right), \tag{A.6}$$

where the updating rule $\mathcal{S} = (\mathcal{S}^r, \mathcal{S}^X, \mathcal{S}^V, \mathcal{S}^\rho)$ is introduced in the next subsection.

(iv). The last step involves drawing parameters from their posterior. First of all, for each index $i \in \mathcal{I}$, draw

$$\Theta_{-K_r}^{(i)} = \left(K_X, \sigma_X^2, K_V, \sigma_V^2, \rho\right)^{(i)} \sim p\left(K_X, \sigma_X^2, K_V, \sigma_V^2, \rho \middle| s_{t+1}^{X,(i)}, s_{t+1}^{V,(i)}, s_{t+1}^{\rho,(i)}\right). \tag{A.7}$$

It remains to draw K_r given $\Theta_{-K_r}^{(i)}$. To draw samples that satisfy the nonnegativity constraint efficiently, we switch from $s_{t+1}^{r,(i)}$ to $\widehat{s}_{t+1}^{r,(i)}$, where $\widehat{s}_{t+1}^{r,(i)}$ is the collection of the sufficient statistics of the minimum entropy posterior for the i-th sample. Further, draw $K_r^{(i)}$ from the minimum entropy posterior

$$K_r^{(i)} \sim p\left(K_r \middle| \widehat{s}_{t+1}^{r,(i)}\right), \tag{A.8}$$

and reject until the expected return given by the i-th sample parameter becomes nonnegative. We then collect all parameters $\Theta^{(i)} = (\Theta_{K_{-r}}, K_r)^{(i)}$ and $(L_{t+1}, s_{t+1}, \Theta)_{i\in\mathcal{I}}^{(i)}$ to form

$$p^N\left(L_{t+1}, s_{t+1}, \Theta \middle| Y^{t+1}\right). \tag{A.9}$$

In the first and second months of any quarter, the likelihood is only associated with the stock return. In the third month of any quarter, the likelihood summarizes the joint likelihood of the stock return and CAY. In either case, both the predictive likelihood and the conditional density of L are jointly Gaussian and the importance sampling procedure is thus straightforward.

A.3 Sufficient Statistics

When the state variable L is realized by each Monte Carlo sample $L^{(i)}$, X, $\ln V$, and r^{ex} follow linear-Gaussian processes, respectively. Imposing the normal prior and normal-inverse-gamma prior gives a closed-form sufficient statistics updating rule.

A.3.1 Excess Return

Suppose we have obtained the sample representation of the filtered state. The stock return scaled by the realized volatility is normally distributed with unit variance

$$\frac{r_{t+1}^{\text{ex}}}{\sqrt{V_t^{(n^{(i)})}}} = K_{r,0}\cdot\frac{1}{\sqrt{V_t^{(n^{(i)})}}} + K_{r,1}\cdot\frac{Z_t}{\sqrt{V_t^{(n^{(i)})}}} + K_{r,1}\cdot\frac{X_t^{(n^{(i)})}}{\sqrt{V_t^{(n^{(i)})}}} + \epsilon_{t+1}^r. \tag{A.10}$$

To simplify the notation, we omit the sample index and write the return dynamics in the following abstract form:

$$\mathbf{Y}_{t+1} = K_r \mathbf{X}_{t+1} + \epsilon_{t+1}, \tag{A.11}$$

where we have

$$\mathbf{X}_{t+1} = \frac{1}{\sqrt{V_t^{(n^{(i)})}}} \cdot \begin{bmatrix} 1 \\ Z_t \\ X_t^{(n^{(i)})} \end{bmatrix}, \quad \mathbf{Y}_{t+1} = \frac{r_{t+1}^{\text{ex}}}{\sqrt{V_t^{(n^{(i)})}}}, \tag{A.12}$$

and ϵ_{t+1} is i.i.d. standard normal. At time t, the conjugate prior of K_r is normal with $K_r \sim \mathcal{N}\big(\mu_t^r, \big(\Omega_t^r\big)^{-1}\big)$. Given $(\mathbf{X}_{t+1}, \mathbf{Y}_{t+1})$, the posterior is also normal with mean and variance updated through:

$$\begin{aligned} \Omega_{t+1}^r &= \Omega_t^r + \mathbf{X}_{t+1}\mathbf{X}_{t+1}^\top, \\ \mu_{t+1}^r &= \left(\mu_t^r \Omega_t^r + \mathbf{Y}_{t+1}\mathbf{X}_{t+1}^\top\right)\left(\Omega_{t+1}^r\right)^{-1}. \end{aligned} \tag{A.13}$$

The set of sufficient statistics for K_r is trivially $s^r = (\mu^r, \Omega^r)$. For CV models, the return variance σ_r^2 is unknown and we impose an inverse-gamma prior. The joint prior for (K_r, σ_r^2) is thus normal-inverse-gamma.

A.3.2 State Variables

With the state variable realized by Monte Carlo samples, the dynamics of X can be analogously written as a generic univariate linear Gaussian process

$$\mathbf{Y}_{t+1} = K_X \mathbf{X}_{t+1} + \sigma_X \epsilon_{t+1}, \tag{A.14}$$

where $(\mathbf{X}_{t+1}, \mathbf{Y}_{t+1})$ is realized by particle learning

$$\mathbf{Y}_{t+1} = X_{t+1}^{(i)}, \quad \mathbf{X}_{t+1} = \begin{bmatrix} 1 & X_t^{(n^{(i)})} \end{bmatrix}. \tag{A.15}$$

At time t, the conjugate prior is normal-inverse-gamma with $(K_X, \sigma_X^2) \sim \mathcal{NIG}(\mu_t^X, \Omega_t^X, S_t^X, d_t^X)$. The posterior has sufficient statistics updated through

$$\begin{aligned}
d_{t+1}^X &= d_t^X + 1, \\
\Omega_{t+1}^X &= \Omega_t^X + \mathbf{X}_{t+1}\mathbf{X}_{t+1}^\top, \\
\mu_{t+1}^X &= \left(\mu_t^X \Omega_t^X + \mathbf{Y}_{t+1}\mathbf{X}_{t+1}^\top\right)\left(\Omega_{t+1}^X\right)^{-1}, \\
S_{t+1}^X &= S_t^X + \mathbf{Y}_{t+1}\mathbf{Y}_{t+1}^\top + \mu_t^X \Omega_t^X \left(\mu_t^X\right)^\top - \mu_{t+1}^X \Omega_{t+1}^X \left(\mu_{t+1}^X\right)^\top .
\end{aligned} \tag{A.16}$$

These sufficient statistics jointly constitute s^X. The sample parameters are drawn from

$$\begin{aligned}
p\left(\sigma_X^2 \middle| s_{t+1}^X\right) &\sim \mathcal{IG}\left(S_{t+1}^X, d_{t+1}^X\right), \\
p\left(K_X \middle| \sigma_X^2, s_{t+1}^X\right) &\sim \mathcal{N}\left(\mu_{t+1}^X, \sigma_X^2 \cdot \left(\Omega_{t+1}^X\right)^{-1}\right),
\end{aligned} \tag{A.17}$$

The posterior updating rule for the dynamics of $\ln V$ follows exactly the same procedure.

A.3.3 Correlation

Consider a sequence of i.i.d. bivariate normal samples $\epsilon_t = (\epsilon_t^r, \epsilon_t^V)$ with zero mean and variance matrix

$$\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}. \tag{A.18}$$

Following Johannes, Korteweg, and Polson (2014), the conjugate prior takes a form that makes the posterior follows

$$p\left(\rho \middle| s_{t+1}^\rho\right) \propto \left(1-\rho^2\right)^{-\frac{t}{2}} \cdot \exp\Big(-\frac{S_{t+1}^{\rho,(1,1)} + S_{t+1}^{\rho,(2,2)} - 2\rho S_{t+1}^{\rho,(1,2)}}{2(1-\rho^2)}\Big), \tag{A.19}$$

where

$$S_{t+1}^\rho = S_t^\rho + \epsilon_{t+1}\epsilon_{t+1}^\top \tag{A.20}$$

defines the sufficient statistics updating mechanism. We draw the sample correlation from the grid $[-0.999, -0.998, ..., 0.999]$, with sampling weights proportional to the density evaluated at these points. To update the correlation for SV and SV-CAY models, we extract $(\epsilon^r_{t+1}, \epsilon^V_{t+1})^{(i)}$ from

$$\left(L_t^{(n^{(i)})}, L_{t+1}^{(i)}, \Theta^{(n^{(i)})}, Y_{t:t+1}\right), \tag{A.21}$$

and update the sufficient statistics through

$$s_{t+1}^{\rho,(i)} = \mathcal{S}^{\rho}\left(s_t^{\rho,(n^{(i)})}, \left(\epsilon^r_{t+1}, \epsilon^V_{t+1}\right)^{(i)}\right), \tag{A.22}$$

The updated correlation samples are drawn from

$$\rho^{(i)} \sim p\left(\rho \middle| s_{t+1}^{\rho,(i)}\right). \tag{A.23}$$

A.3.4 Economic Constraint

This section presents technical details of imposing the constraint of a nonnegative expected return in step (iv) of particle learning. To achieve this goal, we combine the constrained inference approaches of Pettenuzzo, Timmermann, and Valkanov (2014) and Chib and Zeng (2016). At time $t+1$, the investor observes the predictors and forms the forecast of time-$(t+2)$ return from the naïve Bayesian posterior

$$\begin{aligned}
\mathbb{E}_{t+1}\left[r^{\text{ex}}_{t+2}\right] &= \frac{1}{N}\cdot\sum_{i\in\mathcal{I}}\mathbb{E}_{t+1}\left[K_{r,0} + K_{r,1}Z_{t+1} + K_{r,2}X_{t+1}^{(i)}\middle| s_{t+1}^{(i)}\right] \\
&= \frac{1}{N}\cdot\sum_{i\in\mathcal{I}}\left(\mu_{0,t+1}^{r,(i)} + \mu_{1,t+1}^{r,(i)}Z_{t+1} + \mu_{2,t+1}^{r,(i)}X_{t+1}^{(i)}\right).
\end{aligned} \tag{A.24}$$

Inside the bracket is the Bayesian predictive mean given by each sample parameter. Truncating the above expected return boils down to the truncated forecast of Campbell and Thompson (2008). We are particularly interested in embedding the constraint into sequential estimates, i.e., we aim at obtaining parameter posterior that gives a nonnegative expected return for any parameter realization from its support. This can be achieved as follows. For each sample indexed by i, we use an acceptance-rejection scheme to draw parameters satisfying the constraint. A potential drawback is that if the Bayesian predictive mean is negative, the acceptance-rejection scheme may take too many draws to proceed. To alleviate this concern, we use the minimum entropy posterior of Chib and Zeng (2016). The minimum entropy posterior pushes the Bayesian predictive mean above zero and distorts the naïve Bayesian posterior only through that of K_r. Thus, we can start by assuming that $\Theta_{-K_r}^{(i)}$ has already been obtained from $(s_{t+1}^X, s_{t+1}^V, s_{t+1}^\rho)^{(i)}$. Let $\widehat{s}_{t+1}^r$ be the sufficient statistics of the minimum entropy posterior, which differs from s_{t+1}^r only by sufficient statistics controlling the posterior mean of K_r, denoted by $\widehat{\mu}_{t+1}^r$. Defining

$$\mathbf{X}_{t+1} = \begin{bmatrix} 1 \\ Z_{t+1} \\ X_{t+1}^{(n^{(i)})} \end{bmatrix}, \tag{A.25}$$

we can derive $\widehat{\mu}_{t+1}$ from

$$\widehat{\mu}_{t+1}^{r,(i)} = \mu_{t+1}^{r,(i)} \cdot \mathbb{I}_{\mu_{t+1}^{r,(i)}\mathbf{X}_{t+1} \geq 0} + \left(\mu_{t+1}^{r,(i)} - \frac{\mu_{t+1}^{r,(i)}\mathbf{X}_{t+1}}{\mu_{t+1}^{r,(i)}\left(\Omega_{t+1}^{r,(i)}\right)^{-1}\left(\mu_{t+1}^{r,(i)}\right)^\top} \mathbf{X}_{t+1}^\top \left(\Omega_{t+1}^{r,(i)}\right)^{-1} \right) \cdot \mathbb{I}_{\mu_{t+1}^{r,(i)}\mathbf{X}_{t+1} < 0}. \tag{A.26}$$

The above expression implies that the minimum entropy posterior coincides with the naïve Bayesian posterior if the naïve Bayesian predictive mean is nonnegative. It remains to draw $K_r^{(i)} \sim p\big(K_r \big| \widehat{s}_{t+1}^{r,(i)}\big)$ until the expected return

$$K_{r,0}^{(i)} + K_{r,1}^{(i)} Z_{t+1} + K_{r,2}^{(i)} X_{t+1}^{(i)} \tag{A.27}$$

becomes nonnegative. $p\big(K_r \big| (\widehat{s}_{t+1}^r, K_{-r})^{(i)}\big)$ remains multivariate normal. Therefore, the acceptance rate is larger than 50% and the sampling efficiency is thus greatly improved. Our sampling procedure guarantees that the return forecast is positive for any sample parameter in $(\Theta^{(i)})_{i\in\mathcal{I}}$. Finally, the investor uses

$$\frac{1}{N} \cdot \sum_{i \in \mathcal{I}} \left(K_{r,0}^{r,(i)} + K_{r,1}^{r,(i)} Z_{t+1} + K_{r,2}^{r,(i)} X_{t+1}^{(i)} \right) \tag{A.28}$$

to forecast time-$(t+2)$ return. The theoretical effectiveness of the minimum entropy posterior comes from the fact that it minimizes the Kullback-Leibler divergence relative to the naïve Bayesian posterior among all candidate posteriors satisfying the constraint:

$$\begin{aligned} \mathbb{E}_{t+1}\left[K_{r,0}+K_{r,1}Z_{t+1}+K_{r,2}X_{t+1}^{(i)}\middle|\widehat{s}_{t+1}^{(i)}\right] &= \widehat{\mu}_{t+1,0}^{r,(i)}+\widehat{\mu}_{t+1,1}^{r,(i)}Z_{t+1}+\widehat{\mu}_{t+1,2}^{r,(i)}X_{t+1}^{(i)} \\ &\geq 0. \end{aligned} \tag{A.29}$$

Chib and Zeng (2016) show that this is equivalent to solving

$$\operatorname{argmin}_{\widehat{\mu}_{t+1}^{r,(i)}}\left(\widehat{\mu}_{t+1}^{r,(i)}-\mu_{t+1}^{r,(i)}\right)\left(\Omega_{t+1}^{r,(i)}\right)^{-1}\left(\widehat{\mu}_{t+1}^{r,(i)}-\mu_{t+1}^{r,(i)}\right)^{\top}, \quad \text{subject to} \quad \widehat{\mu}_{t+1}^{r,(i)}\mathbf{X}_{t+1}\geq 0. \tag{A.30}$$

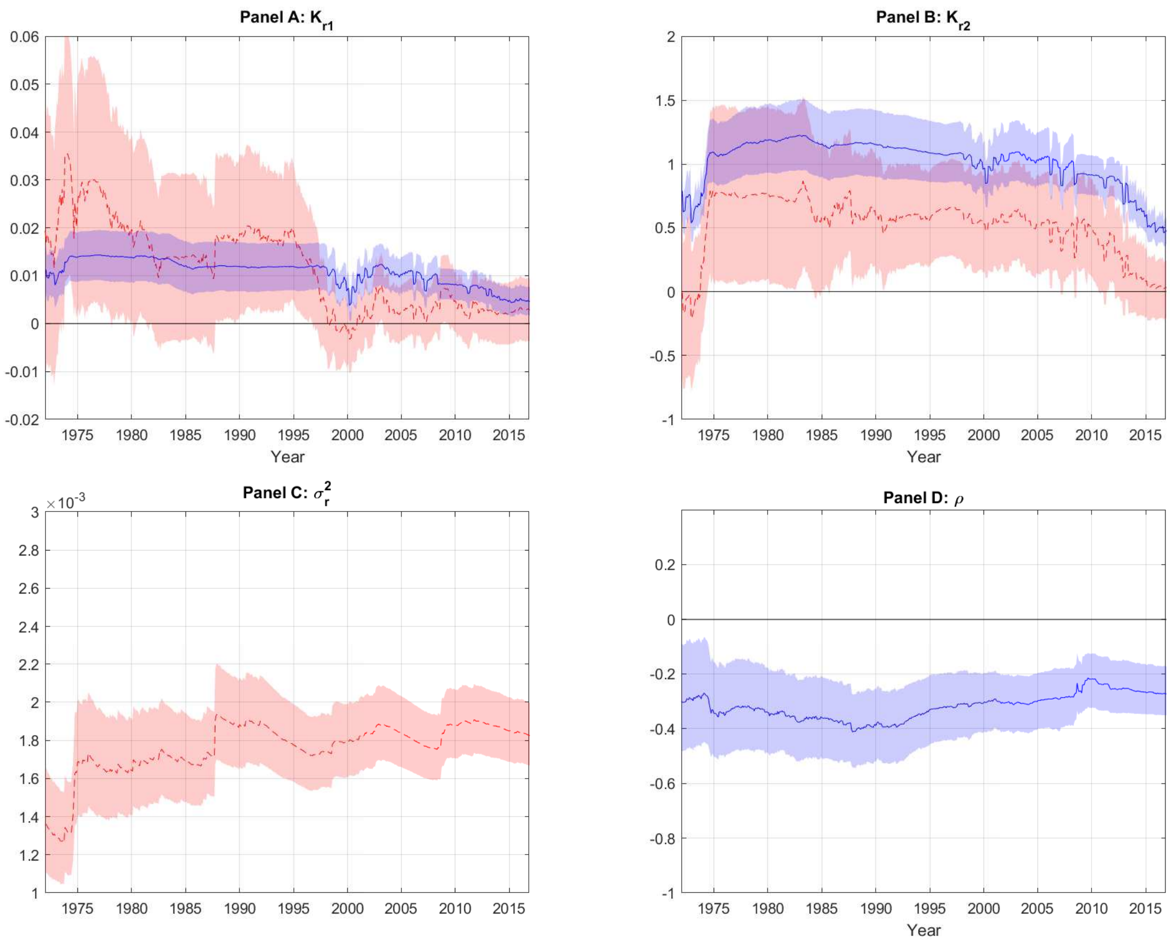

Figure 1: Sequential estimates of CV-DP-CAY and SV-DP-CAY models. Solid line corresponds to the posterior mean. Colored areas correspond to the 90%-credible interval. The sample period extends from January 1972 to December 2016.

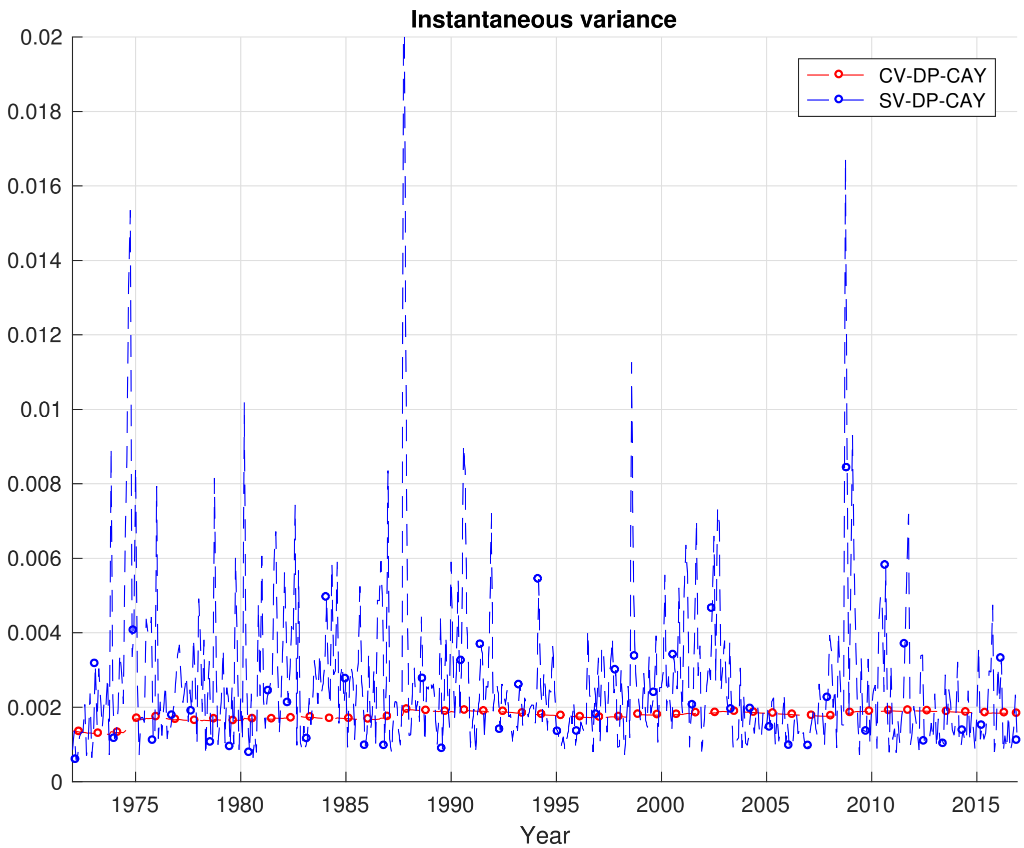

Figure 2: Instantaneous variance given by CV-DP-CAY and SV-DP-CAY models. At each time, the instantaneous variance is the posterior mean of the filtered variance.

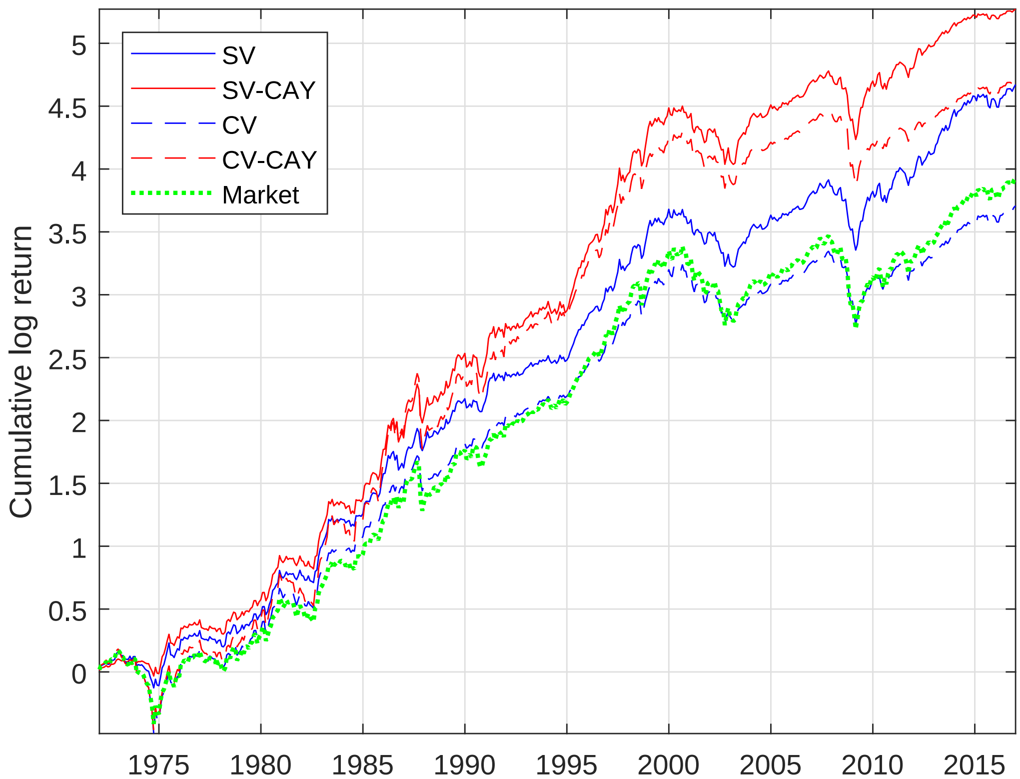

Figure 3: Portfolio wealth averaged across different model specifications. For each model specification (CV, CV-CAY, SV, and SV-CAY), we plot the monthly evolution of the average the wealth across 15 different strategies, 14 strategies using financial predictors and one model without using a financial predictor. The investment period starts in January 1972 and ends in December 2016.

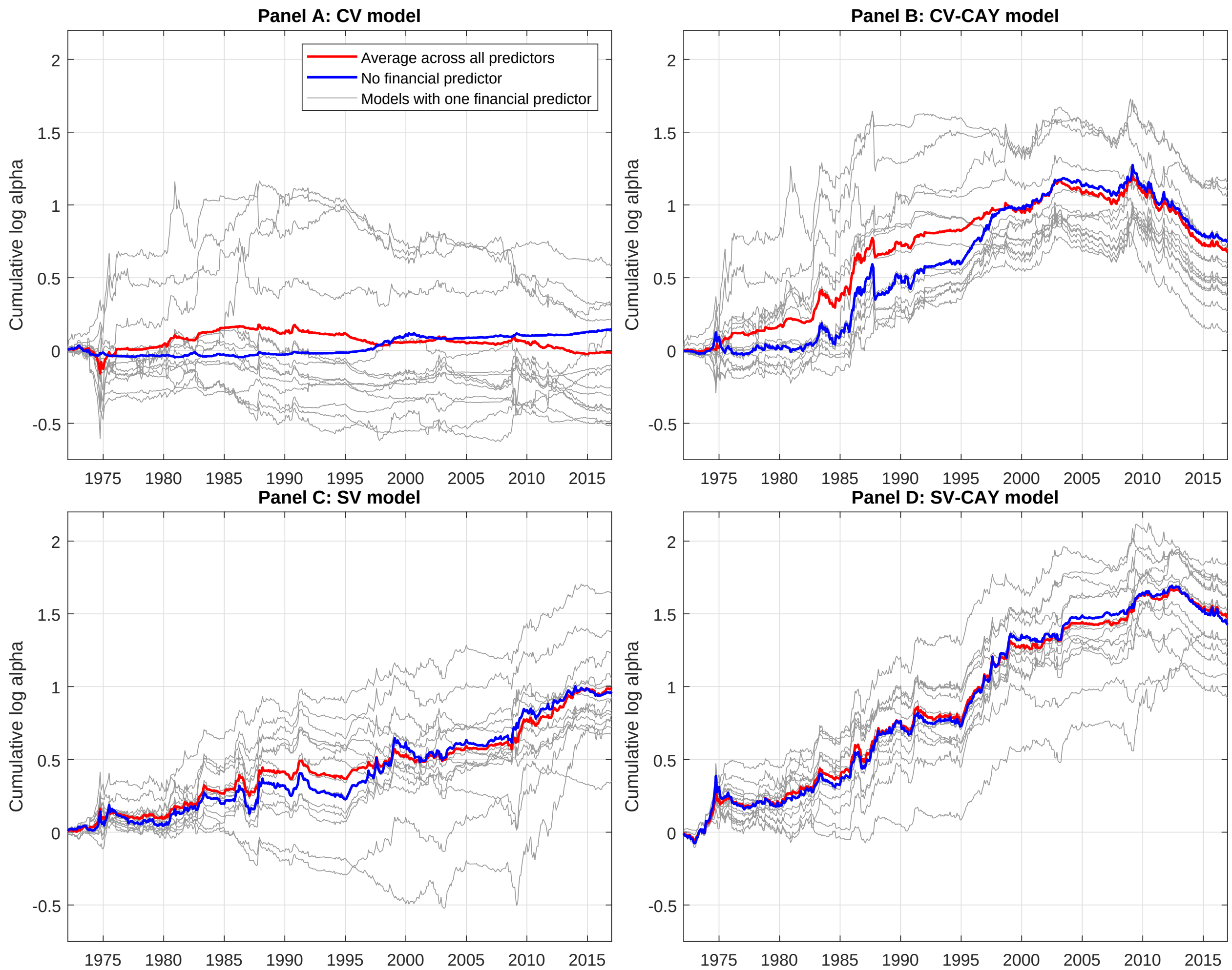

Figure 4: Cumulative alphas for different strategies and models. In Panels A to D, we plot for each model specification the cumulative alphas relative to the market portfolio. The gray lines represent the trajectories for each of the 14 predictors. Blue line denotes the specification, when no predictor is used. Red lines denote the average across all of the 15 specifications.

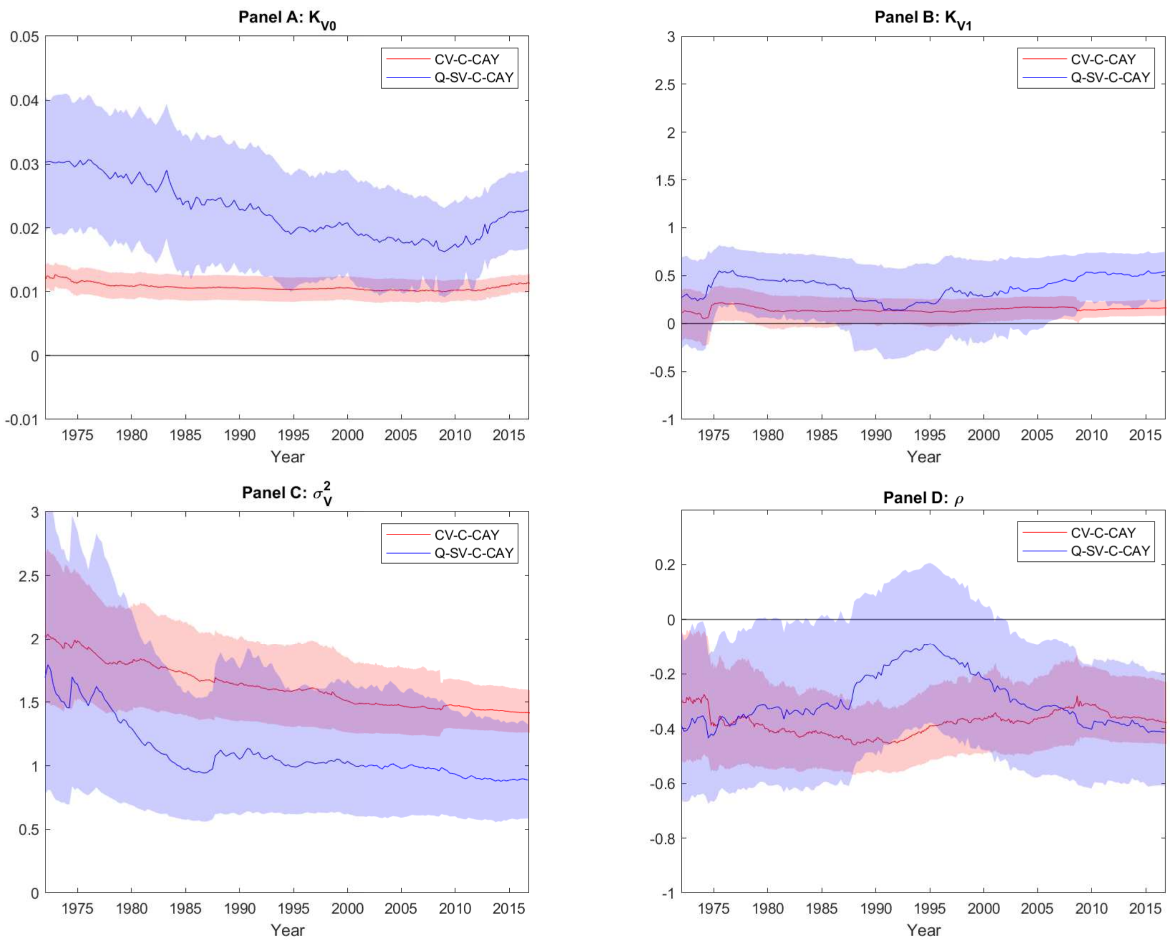

Figure 5: Sequential estimates of SV-C-CAY and Q-SV-C-CAY models. Solid line corresponds to the posterior mean. Colored areas correspond to the 90%-credible interval. The sample period extends from January 1972 to December 2016.

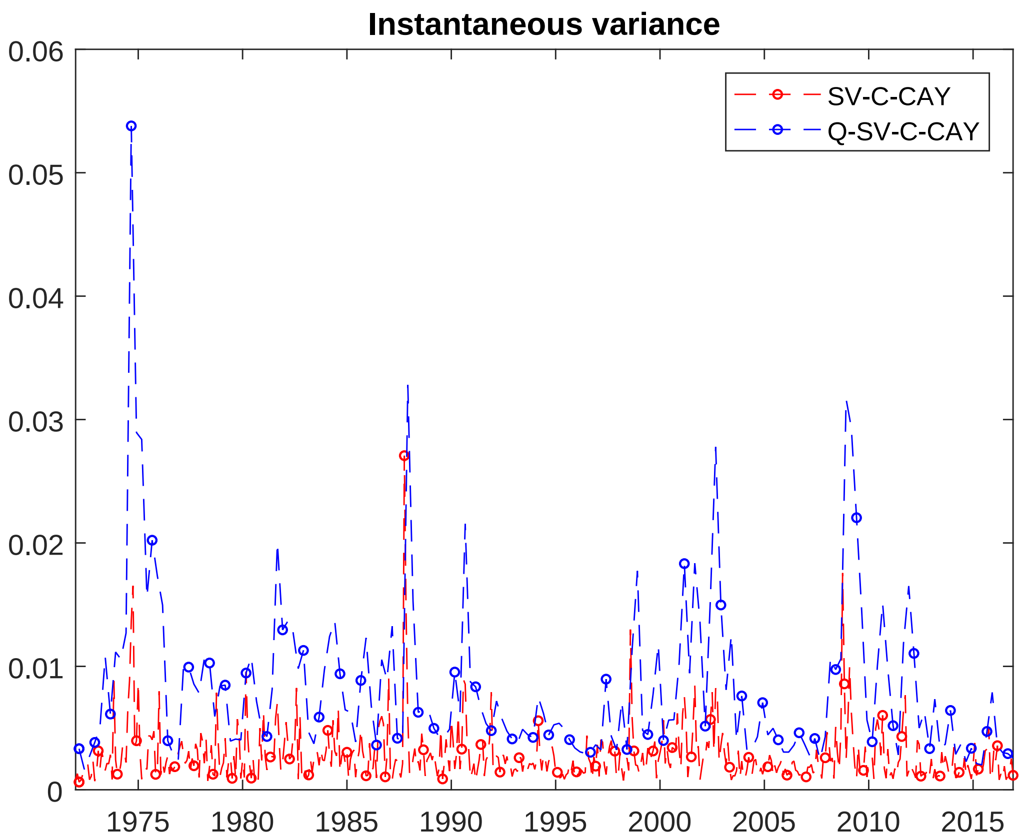

Figure 6: Instantaneous variance given by SV-C-CAY and Q-SV-C-CAY models. At each time, the instantaneous variance is the posterior mean of the filtered variance.

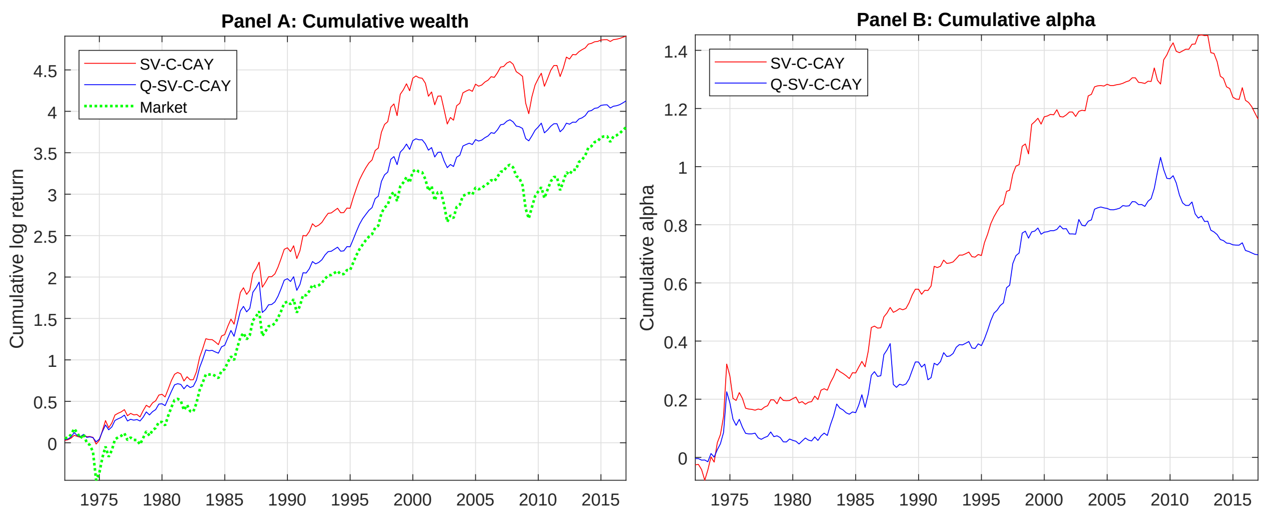

Figure 7: Portfolio strategies SV-C-CAY and Q-SV-C-CAY with quarterly rebalancing. In Panel A, we plot the log of cumulative wealth, together with the evolution of the market portfolio. In Panel B, we plot the cumulative alphas relative to the market. The investment period starts in January 1972 and ends in December 2016 and portfolios are rebalanced at the end of each quarter. Risk aversion is set to $\gamma = 4$.

Table 1: Summary Statistics of Returns and Predictors

	Mean	St.dev	Skew	Kurt
Index return (r)	0.103	0.145	−0.636	2.475
Risk-free rate (r^f)	0.044	0.009	0.835	1.063
Excess return (r^{ex})	0.059	0.145	−0.654	2.434
DP	−3.533	0.404	−0.316	−0.627
DY	−3.527	0.405	−0.319	−0.606
EP	−2.802	0.418	−0.824	3.192
DE	−0.731	0.297	2.594	15.488
SVAR	0.002	0.004	10.870	154.017
BM	0.518	0.247	0.601	−0.313
NTIS	0.013	0.019	−0.968	0.934
TBL	0.044	0.031	0.844	1.090
LTY	0.061	0.027	0.818	0.211
LTR	0.005	0.028	0.492	3.137
TMS	0.017	0.014	−0.161	−0.156
DFY	0.010	0.004	1.790	4.546
DFR	−0.055	0.036	−0.598	2.457
INFL	0.003	0.004	0.063	2.740
CAY	0.000	0.024	−0.363	−0.337

Summary statistics of returns and predictors. We report the sample mean (Mean), the standard deviation (Std.dev), skewness (Skew), and excess kurtosis (Kurt). All returns are logarithmic and annualized. As predictors, we use dividend-price ratio (DP), dividend yield (DY), earning-price ratio (EP), dividend-payout ratio (DE), stock variance (SVAR), book-to-market ratio (BM), net equity expansion (NTIS), treasury-bill rate (TBL), long-term yield (LTY), long-term rate of return (LTR), term spread (TMS), default yield spread (DFY), default return spread (DFR), inflation (INFL), and consumption-wealth ratio (CAY). CAY is real-time, quarterly sampled and demeaned. The construction follows Welch and Goyal (2007). Other variables are monthly sampled. The sample period extends from 1952:Q1 to 2016:Q4.

Table 2: Model Estimates I

	CV Models $K_{r,0}$	CV Models $K_{r,1}$	SV Models $K_{r,0}$	SV Models $K_{r,1}$
C	0.40		0.88	
	(0.16)		(0.10)	
DP	1.70	0.36	3.43	0.75
	(1.45)	(0.40)	(0.78)	(0.24)
DY	1.93	0.42	4.30	1.00
	(1.45)	(0.40)	(0.54)	(0.16)
EP	1.04	0.22	1.87	0.34
	(1.07)	(0.37)	(0.76)	(0.28)
DE	0.53	0.17	1.86	1.58
	(0.43)	(0.54)	(0.24)	(0.36)
SVAR	0.64	−111.32	1.06	−47.94
	(0.18)	(37.59)	(0.08)	(53.25)
BM	0.40	0.02	0.05	2.17
	(0.34)	(0.62)	(0.31)	(0.43)
NTIS	0.48	−6.92	1.06	−6.60
	(0.18)	(7.91)	(0.15)	(5.70)
TBL	0.79	−8.26	1.94	−53.58
	(0.29)	(5.14)	(0.11)	(4.05)
LTY	0.76	−5.39	2.17	−21.83
	(0.41)	(5.92)	(0.11)	(3.13)
LTR	0.51	3.10	0.60	8.66
	(0.15)	(3.85)	(0.03)	(1.38)
TMS	0.01	21.47	0.47	29.91
	(0.26)	(11.21)	(0.13)	(7.61)
DFY	0.05	35.46	0.77	15.45
	(0.39)	(36.14)	(0.24)	(28.86)
DFR	1.22	12.90	1.44	13.02
	(0.31)	(4.22)	(0.11)	(2.99)
INFL	0.43	−8.73	1.17	−99.45
	(0.21)	(45.48)	(0.03)	(26.38)

Estimates of predictive coefficients for CV and SV models. We report the posterior mean and standard errors (in parentheses) obtained in the last month of the sample period. For ease of display, mean and standard errors are reported in percentage.

Table 3: Model Estimates II

	CV-CAY Models			SV-CAY Models		
	$K_{r,0}$	$K_{r,1}$	$K_{r,2}$	$K_{r,0}$	$K_{r,1}$	$K_{r,2}$
C	0.49		7.72	1.26		52.73
	(0.15)		(12.93)	(0.06)		(6.80)
DP	1.53	0.29	3.44	2.93	0.46	47.87
	(1.44)	(0.40)	(13.73)	(0.64)	(0.18)	(5.79)
DY	1.81	0.37	2.71	3.91	0.67	58.95
	(1.44)	(0.40)	(13.66)	(0.49)	(0.15)	(5.88)
EP	0.89	0.14	4.97	2.48	0.40	52.63
	(1.05)	(0.36)	(13.58)	(0.60)	(0.21)	(6.25)
DE	0.63	0.20	7.48	1.49	0.50	48.92
	(0.42)	(0.53)	(13.15)	(0.19)	(0.31)	(5.59)
SVAR	0.73	−117.70	14.17	1.25	−56.44	51.77
	(0.17)	(37.51)	(13.11)	(0.06)	(35.18)	(5.97)
BM	0.51	−0.06	6.34	0.56	1.91	47.80
	(0.33)	(0.60)	(13.70)	(0.21)	(0.37)	(6.91)
NTIS	0.56	−8.76	16.17	1.99	−28.97	67.43
	(0.17)	(7.32)	(15.77)	(0.13)	(5.90)	(11.54)
TBL	1.04	−12.38	29.30	2.07	−32.34	80.45
	(0.28)	(5.11)	(15.67)	(0.08)	(2.58)	(7.78)
LTY	1.21	−11.58	27.15	4.97	−83.00	128.72
	(0.43)	(6.40)	(16.70)	(0.09)	(2.27)	(6.66)
LTR	0.51	4.00	−7.39	1.22	3.57	37.94
	(0.16)	(4.87)	(16.42)	(0.07)	(3.06)	(9.16)
TMS	0.08	21.29	7.32	0.81	81.10	83.15
	(0.26)	(11.09)	(14.13)	(0.11)	(6.37)	(11.86)
DFY	0.25	25.43	6.33	1.32	−36.55	38.51
	(0.39)	(35.77)	(13.13)	(0.12)	(21.52)	(8.57)
DFR	1.26	11.86	−1.07	1.83	11.56	37.48
	(0.30)	(4.25)	(13.29)	(0.10)	(2.19)	(5.53)
INFL	0.56	−21.75	9.04	1.19	−98.22	44.17
	(0.20)	(44.58)	(13.21)	(0.06)	(25.74)	(6.22)

Estimates of predictive coefficients for CV-CAY and SV-CAY models. We report the posterior mean and standard errors (in parentheses) obtained in the last month of the sample period. For ease of display, mean and standard errors are reported in percentage.

Table 4: Summary Statistics of Portfolio Returns for CV and SV Models

	Mean (%)	St.dev (%)	Skew	Kurt	SR	CER (%)
			Panel A: CV models			
C	$3.314^{***}_{\diamond\diamond\diamond}$	$8.945^{***}_{\diamond\diamond\diamond}$	-0.638^{**}	2.232	0.370^{**}	1.688^{***}
DP	$3.553^{***}_{\diamond\diamond}$	14.548	-0.057	8.917	$0.244^{**}_{\diamond\diamond}$	$-0.784^{***}_{\diamond\diamond\diamond}$
DY	3.739^{**}	$15.109_{\diamond\diamond}$	-0.053	8.501	$0.247^{**}_{\diamond\diamond}$	$-0.949^{***}_{\diamond\diamond\diamond}$
EP	$3.063^{***}_{\diamond\diamond\diamond}$	13.373	-0.049	14.674	$0.229^{***}_{\diamond\diamond\diamond}$	$-0.635^{***}_{\diamond\diamond\diamond}$
DE	3.834^{**}	$13.662_{\diamond\diamond}$	-0.421	7.029^{**}	0.281^{**}	$-0.059^{***}_{\diamond\diamond\diamond}$
SVAR	$2.752^{***}_{\diamond\diamond\diamond}$	12.287	-0.764	$22.730^{**}_{\diamond\diamond}$	$0.224^{**}_{\diamond}$	$-0.601^{***}_{\diamond\diamond\diamond}$
BM	$3.535^{**}_{\diamond\diamond}$	14.017	0.361	12.571	$0.252^{**}_{\diamond\diamond}$	$-0.403^{***}_{\diamond\diamond\diamond}$
NTIS	$5.764^{**}_{\diamond\diamond}$	$16.797^{**}_{\diamond\diamond}$	-0.127	$6.380^{***}_{\diamond\diamond\diamond}$	$0.343_{\diamond}$	$-0.155^{***}_{\diamond\diamond\diamond}$
TBL	4.195^{***}	$13.582_{\diamond\diamond\diamond}$	-0.608	$2.517_{\diamond\diamond}$	0.309^{**}	$0.382^{***}_{\diamond\diamond\diamond}$
LTY	$3.668^{***}_{\diamond}$	11.745	-0.389	3.682	0.312^{**}	$0.878^{***}_{\diamond\diamond\diamond}$
LTR	5.980^{*}	$16.986_{\diamond}$	0.427	$5.678^{***}_{\diamond\diamond\diamond}$	0.352^{**}	$0.262^{***}_{\diamond\diamond\diamond}$
TMS	7.122^{*}	18.266	-1.373	15.992	0.390	$-2.127^{***}_{\diamond\diamond\diamond}$
DFY	5.561^{*}	16.014	0.715	$8.230^{***}_{\diamond\diamond\diamond}$	0.347	$0.562^{***}_{\diamond\diamond\diamond}$
DFR	5.383^{**}	$16.162_{\diamond\diamond\diamond}$	-0.493	4.158	$0.333^{***}_{\diamond\diamond}$	$-0.118^{***}_{\diamond\diamond\diamond}$
INFL	$3.398^{***}_{\diamond\diamond\diamond}$	$12.601^{**}_{\diamond\diamond}$	-0.733	$6.565_{\diamond}$	$0.270^{**}_{\diamond\diamond}$	$0.035^{***}_{\diamond\diamond\diamond}$
			Panel B: SV models			
C	7.038^{*}	15.213^{***}	-0.252	0.572^{*}	0.463	2.362^{***}
DP	6.151^{**}	13.212^{**}	0.223	1.983	0.466	2.735^{***}
DY	5.086^{***}	11.778^{***}	0.150	2.172	0.432^{*}	2.366^{***}
EP	6.951^{*}	14.521	-0.195	0.570	0.479	2.721^{***}
DE	3.911^{***}	10.765^{***}	0.065	6.351^{*}	0.363^{**}	1.575^{**}
SVAR	6.310^{*}	14.236^{***}	-0.276	0.348	0.443	2.222^{***}
BM	6.014	13.986	0.152	1.894	0.430	2.177^{***}
NTIS	9.265	18.864	-0.262	0.941	0.491	1.977
TBL	4.594^{**}	10.615^{***}	-0.111	5.981^{**}	0.433	2.324^{***}
LTY	6.032	13.134	-0.320	2.115	0.459	2.519^{***}
LTR	7.143	15.471	-0.138	1.965	0.462	2.284^{***}
TMS	9.023	16.696^{***}	-0.349	1.974	0.540	3.239
DFY	6.698^{***}	14.986^{***}	-0.287	0.558	0.447	2.153^{***}
DFR	6.706^{**}	13.060^{***}	-0.273^{*}	2.138	0.513	3.238^{***}
INFL	6.455	14.697	-0.426	1.787	0.439	1.996^{***}
			Panel C: Benchmark models			
PM	2.747	7.692	-0.892	3.956	0.357	1.539
BH	6.312	15.171	-0.447	1.981	0.416	1.560

Summary statistics of portfolio returns for CV (Panel A) and SV models (Panel B) for $\gamma = 4$. We report the sample mean of simple excess returns (Mean), the standard deviation (Std.dev), skewness (Skew), excess kurtosis (Kurt), Sharpe ratio (SR), and certainty equivalent return (CER). Mean and Std.dev are annualized and reported in percentage. In Panel C, we report two benchmarks. The prevailing-mean model (PM) assumes a constant expected return and standard deviation and is estimated by all data available at a given point in time. PM assumes no estimation risk for portfolio optimization. We also report the results for the buy-and-hold strategy (BH). For the significance tests, we rely on Ledoit and Wolf (2018). By ***, **, and *, we denote significance levels of $1\%, 5\%$, and 10%, respectively, when benchmmarking against SV-CAY models. By $^{\diamond\diamond\diamond}$, $^{\diamond\diamond}$, $^{\diamond}$, we denote the corresponding significance levels when benchmarking the CV against the SV models in Panel A. The investment period starts in January 1972 and ends in December 2016.

Table 5: Summary Statistics of Portfolio Returns for CV-CAY and SV-CAY Models

	Mean (%)	St.dev (%)	Skew	Kurt	SR	CER (%)
			Panel A: CV-CAY models			
C	6.725	15.873	−1.283	13.267	0.424	$0.562^{***}_{\diamond\diamond\diamond}$
DP	6.230^{*}	$16.771^{**}_{\diamond\diamond\diamond}$	−0.128	4.661^{*}	0.372^{**}	$0.441^{***}_{\diamond\diamond\diamond}$
DY	6.371	$16.884^{**}_{\diamond\diamond\diamond}$	−0.040	4.531^{*}	0.377^{**}	$0.555^{***}_{\diamond\diamond\diamond}$
EP	6.546	$17.194^{*}_{\diamond}$	−0.512	7.745	0.381^{**}	$0.034^{***}_{\diamond\diamond\diamond}$
DE	$6.976_{\diamond}$	$18.427^{***}_{\diamond\diamond\diamond}$	−0.816	9.806	0.379	$-1.115^{***}_{\diamond\diamond\diamond}$
SVAR	5.883	17.699	−0.929	12.747	0.332	$-1.658^{***}_{\diamond\diamond\diamond}$
BM	6.419	$18.207^{***}_{\diamond\diamond}$	0.008	7.848	0.353	$-0.515^{***}_{\diamond\diamond\diamond}$
NTIS	7.247	18.566	−0.351	8.610	0.390	$-0.482^{***}_{\diamond\diamond\diamond}$
TBL	7.314	$18.415^{***}_{\diamond\diamond\diamond}$	−1.006	6.923	0.397	$-0.581^{***}_{\diamond\diamond\diamond}$
LTY	7.455	$17.649^{***}_{\diamond\diamond\diamond}$	−0.619	5.442	0.422^{*}	$0.639^{***}_{\diamond\diamond\diamond}$
LTR	8.983	$20.569^{***}_{\diamond\diamond\diamond}$	0.412	$4.913^{**}_{\diamond\diamond}$	0.437	$0.562^{***}_{\diamond\diamond\diamond}$
TMS	9.281	$21.684^{**}_{\diamond\diamond}$	−1.115	16.732	0.428	$-6.381^{***}_{\diamond\diamond\diamond}$
DFY	8.593	$22.186^{***}_{\diamond\diamond\diamond}$	0.343	$7.722^{***}_{\diamond\diamond\diamond}$	0.387	$-1.717^{***}_{\diamond\diamond\diamond}$
DFR	8.990	$19.895^{***}_{\diamond\diamond\diamond}$	−0.224	2.744	0.452^{*}	$0.762^{***}_{\diamond\diamond\diamond}$
INFL	6.410	$17.169^{**}_{\diamond}$	−0.697	7.097	0.373	$-0.174^{***}_{\diamond\diamond\diamond}$
			Panel B: SV-CAY models			
C	$8.561^{\bullet\bullet\bullet}_{\circ\circ\circ}$	$16.717^{\bullet\bullet\bullet}_{\circ\circ}$	$-0.309^{\bullet\bullet}_{\circ}$	1.759	$0.512^{\bullet\bullet}_{\circ\circ\circ}$	$2.800^{\bullet\bullet\bullet}_{\circ\circ\circ}$
DP	$8.217^{\bullet\bullet\bullet}_{\circ\circ\circ}$	$14.846^{\bullet\bullet\bullet}$	$-0.008^{\bullet\bullet\bullet}_{\circ\circ\circ}$	2.167	$0.553^{\bullet}_{\circ\circ\circ}$	$3.797^{\bullet\bullet\bullet}_{\circ\circ\circ}$
DY	$8.265^{\bullet\bullet\bullet}_{\circ\circ}$	$14.891^{\bullet\bullet\bullet}$	$-0.004^{\bullet\bullet\bullet}_{\circ\circ\circ}$	3.322	$0.555^{\bullet}_{\circ\circ}$	$3.788^{\bullet\bullet\bullet}_{\circ\circ\circ}$
EP	$8.306^{\bullet\bullet\bullet}_{\circ\circ\circ}$	$14.980^{\bullet\bullet\bullet}$	$-0.137^{\bullet\bullet\bullet}_{\circ\circ\circ}$	2.073	$0.554^{\bullet\bullet}_{\circ\circ\circ}$	$3.760^{\bullet\bullet\bullet}_{\circ\circ\circ}$
DE	$7.137^{\bullet\bullet\bullet}_{\circ}$	$14.145^{\bullet\bullet\bullet}_{\circ}$	$-0.217^{\bullet\bullet}_{\circ\circ}$	3.106	$0.505^{\bullet}_{\circ\circ}$	$3.035^{\bullet\bullet\bullet}_{\circ\circ\circ}$
SVAR	$7.890^{\bullet\bullet\bullet}_{\circ\circ}$	$15.901^{\bullet\bullet\bullet}$	$-0.219^{\bullet\bullet}_{\circ\circ}$	$1.805^{\bullet}$	$0.496^{\bullet}_{\circ\circ}$	$2.733^{\bullet\bullet\bullet}_{\circ\circ\circ}$
BM	$6.389^{\bullet\bullet}$	$14.248^{\bullet\bullet\bullet}$	$0.103^{\bullet\bullet\bullet}_{\circ\circ\circ}$	2.831	0.448	$2.353_{\circ\circ\circ}$
NTIS	$8.663^{\bullet\bullet\bullet}_{\circ\circ\circ}$	$18.954^{\bullet\bullet\bullet}_{\circ\circ\circ}$	−0.514	1.778	$0.457_{\circ\circ}$	$1.047_{\circ\circ}$
TBL	$7.513^{\bullet\bullet\bullet}_{\circ}$	$14.953^{\bullet\bullet\bullet}$	−0.374	2.845	$0.502^{\bullet}_{\circ}$	$2.874^{\bullet\bullet\bullet}_{\circ\circ\circ}$
LTY	$7.678^{\bullet\bullet\bullet}_{\circ}$	$13.195^{\bullet\bullet\bullet}_{\circ\circ}$	$0.018^{\bullet\bullet}_{\circ\circ}$	3.187	$0.582^{\bullet\bullet}_{\circ\circ}$	$4.180^{\bullet\bullet\bullet}_{\circ\circ\circ}$
LTR	$8.299^{\bullet\bullet\bullet}_{\circ\circ\circ}$	$15.468^{\bullet\bullet\bullet}$	$-0.085^{\bullet\bullet}_{\circ\circ}$	1.820	$0.537^{\bullet\bullet}_{\circ\circ\circ}$	$3.469^{\bullet\bullet\bullet}_{\circ\circ\circ}$
TMS	$9.483^{\bullet\bullet\bullet}_{\circ\circ\circ}$	$17.689^{\bullet\bullet\bullet}_{\circ\circ\circ}$	$-0.384^{\bullet}_{\circ}$	2.582	$0.536^{\bullet\bullet}_{\circ\circ\circ}$	$2.907^{\bullet\bullet\bullet}_{\circ\circ\circ}$
DFY	$8.971^{\bullet\bullet\bullet}_{\circ\circ\circ}$	$17.588^{\bullet\bullet\bullet}_{\circ\circ\circ}$	$-0.313^{\bullet\bullet}_{\circ}$	$1.502^{\bullet}$	$0.51^{\bullet}_{\circ\circ}$	$2.592^{\bullet\bullet\bullet}_{\circ\circ\circ}$
DFR	$8.654^{\bullet\bullet\bullet}_{\circ\circ\circ}$	$14.928^{\bullet\bullet\bullet}$	$-0.019^{\bullet}_{\circ}$	2.059	$0.58^{\bullet\bullet}_{\circ\circ\circ}$	$4.175^{\bullet\bullet\bullet}_{\circ\circ\circ}$
INFL	$7.113^{\bullet\bullet\bullet}$	$14.838^{\bullet\bullet\bullet}$	$-0.323^{\bullet}$	2.831	$0.479_{\circ}$	$2.565^{\bullet\bullet\bullet}_{\circ\circ\circ}$

Summary statistics of portfolio returns for CV-CAY (Panel A) and SV-CAY models (Panel B) for $\gamma = 4$. We report the sample mean of simple excess returns (Mean), the standard deviation (Std.dev), skewness (Skew), excess kurtosis (Kurt), Sharpe ratio (SR), and certainty equivalent return (CER). Mean and Std.dev are annualized and reported in percentage. In Panel A, for the significance tests for the different performance measures relative to the SV-CAY model, we rely on the robust test statistics of Ledoit and Wolf (2018). In Panel A, by ***, **, and *, we denote significance levels of 1%, 5%, and 10%, respectively, when testing against SV-CAY, and by $^{\diamond\diamond\diamond}, ^{\diamond\diamond}, ^{\diamond}$ the corresponding levels when testing against SV. In Panel B, we use for corresponding significance tests of the SV-CAY models against the benchmark models PM and BH in Table 4, Panel C, by $^{\bullet\bullet\bullet}, ^{\bullet\bullet}, ^{\bullet}$ and $^{\circ\circ\circ}, ^{\circ\circ}, ^{\circ}$, respectively, as markers. The investment period starts in January 1972 and ends in December 2016.

Table 6: Alpha tests for different model specifications and predictors

	CV		CV-CAY		SV		SV-CAY	
	alpha (%)	p-value	alpha (%)	p-value	alpha (%)	p-value	alpha (%)	p-value
C	0.335	0.110	1.887	0.104	2.323**	0.024	3.386***	0.004
DP	−0.722	0.577	1.039	0.352	2.225*	0.053	3.819***	0.002
DY	−0.625	0.657	1.206	0.311	1.688	0.121	4.070***	0.005
EP	−0.988	0.431	1.177	0.296	2.386**	0.014	3.801***	0.002
DE	−0.458	0.599	1.251	0.275	0.980	0.412	2.888***	0.008
SVAR	−0.885	0.337	0.593	0.683	1.878**	0.020	3.042**	0.013
BM	−0.728	0.451	0.899	0.494	1.811	0.132	2.338*	0.064
NTIS	0.787	0.569	1.675	0.235	3.310***	0.002	2.589***	0.007
TBL	−0.193	0.775	1.396*	0.084	2.081	0.134	3.219**	0.018
LTY	−0.167	0.744	1.783**	0.042	2.317*	0.061	4.098***	0.001
LTR	1.057	0.386	2.932*	0.066	2.648*	0.063	3.591***	0.003
TMS	1.620	0.245	2.907	0.109	3.945***	0.001	4.062***	0.001
DFY	1.042	0.494	2.177	0.242	2.018**	0.026	3.516***	0.008
DFR	0.535	0.609	2.904**	0.018	2.990***	0.007	4.356***	0.001
INFL	−0.603	0.428	1.043	0.283	2.047*	0.066	2.806**	0.017

We report the annualized alphas and their p-values for different model specifications and predictors. The test statistics for the alpha are based on the robust method of Leippold and Rüegg (2018). By ***, **, and *, we denote significance levels of 1%, 5%, and 10%, respectively. The investment period starts in January 1972 and ends in December 2016.

Table 7: Summary Statistics of Portfolio Returns of Mixed-Frequency and Quarterly Models

	$\overline{w}$	Mean (%)	St.dev (%)	Skew	Kurt	SR	CER (%)	alpha (%)
				Panel A: $\gamma = 2$				
SV-C-CAY	1.552	11.811	26.302	-0.371	1.283	0.449	4.397	$3.944^{\bullet\bullet}$
Q-SV-C-CAY	1.297^{***}	8.771^{**}	21.107^{***}	-0.712	3.160	0.416	3.800^{***}	2.602
				Panel B: $\gamma = 4$				
SV-C-CAY	0.948	7.657	16.690	-0.257	1.660	0.489	1.750	$2.751^{\bullet\bullet}$
Q-SV-C-CAY	0.767^{***}	5.311^{**}	12.723^{***}	-0.693	4.177	0.417	1.728^{***}	1.689
				Panel C: $\gamma = 6$				
SV-C-CAY	0.681	5.619	11.960	-0.225	1.664	0.469	1.109	$2.134^{\bullet\bullet}$
Q-SV-C-CAY	0.552^{***}	3.838^{**}	9.319^{***}	-0.844	5.620	0.412	0.875^{***}	1.245
				Panel D: $\gamma = 8$				
SV-C-CAY	0.525	4.351	9.196	-0.219	1.654	0.473	0.814	$1.680^{\bullet\bullet}$
Q-SV-C-CAY	0.426^{***}	2.949^{**}	7.255^{*}	-0.924	6.251	0.406	0.542^{***}	0.949

Summary statistics of portfolio returns for mixed-frequency models and quarterly models using CAY as predictor. We report the sample mean of simple excess returns (Mean), the standard deviation (Std.dev), skewness (Skew), excess kurtosis (Kurt), Sharpe ratio (SR), and certainty equivalent return (CER). Mean and Std.dev are annualized and reported in percentage. The portfolio is rebalanced quarterly in accordance with the quarterly preference. $\overline{w}$ is the average portfolio weight. Returns are sampled at quarterly frequency. Moments and Sharpe ratios are annualized. Q corresponds to quarterly-evolving models. C refers to models without monthly financial predictors. For the significance tests of the performance measures of the Q-SV-C-CAY to the SV-C-CAY strategy, we rely on Ledoit and Wolf (2018). By ***, **, and *, we denote significance levels of $1\%, 5\%$, and 10%, respectively. We also test for the significance of the alpha relative to the market using Leippold and Rüegg (2018) and denote the respective significance levels by $^{\bullet\bullet\bullet}, ^{\bullet\bullet}, ^{\bullet}$. The investment period starts in the first quarter of 1972 and ends in the fourth quarter of 2016.